PHYSICS 200C, SPRING 2016
ELECTRICITY AND MAGNETISM

Assignment Three, Due Friday, April 28, 5:00 pm.

[1.] Show the fields

$$\vec{\mathbf{E}} = E_0 \cos\omega t \cos kx \cos ky \,\hat{\mathbf{z}}$$
$$\vec{\mathbf{B}} = B_0 \sin\omega t \Big(\cos kx \sin ky \,\hat{\mathbf{x}} - \sin kx \cos ky \,\hat{\mathbf{y}} \Big)$$

satisfy the fundamental laws of electricity and magnetism in vacuum assuming $E_0 = \alpha B_0$ and $\omega = \beta c k$ where α, β are certain constants. Find these constants. These fields can exist inside a metal box of dimension π/k in the x and y directions, and arbitrary "height" in the z direction. Sketch what $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ look like in a cross section of the box perpendicular to $\hat{\mathbf{z}}$.

[2.] In class we treated a metal as a cloud of *noninteracting* electrons. We commented that this seemingly outrageous viewpoint is not so bizarre if we understand that the Pauli principle causes the electron kinetic energy to be really huge. Take the equation from class relating the number of electrons N to the largest occupied momentum magnitude $k_{\rm F}$

$$N = 2 \sum_{\vec{k}}^{|\vec{k}|<k_{\rm F}} = 2 \frac{V}{(2\pi)^3} \int d^3k = 2 \frac{V}{(2\pi)^3} \frac{4}{3} \pi k_{\rm F}^3$$

Look up a typical value for the density N/V of electrons in a metal. Use this value to compute $k_{\rm F}$ and then compare the kinetic energy $\hbar^2 k_{\rm F}^2 / 2m_e$ with the interaction energy $e^2/(4\pi\epsilon_0 b)$ where a typical electron separation b is given by $nb^3 = 1$.

Comment 1: Better than looking the density of electrons up would be to estimate it from 'basic chemistry': (i) A typical solid has a (mass) density of a few grams per cm^3; (ii) A typical metal has a few electrons in its outermost shell to form the free electron cloud, the remainder are bound to the nucleus; (iii) A mole has $N_A = 6 \times 10^{23}$ atoms and the mass of each is the number of protons plus the number of neutrons (six for lithium, for example) times their mass $m = 1.67 \times 10^{-24}$ g. (Is it a coincidence that $N_A \times m_p = 1$?)

Comment 2: A famous problem in astrophysics is based on the exact same considerations: Take a typical mass of a neutron star (a few solar masses) and volume. (A neutron star has a radius smaller than Davis, $R \approx 10$ km!) Compute k_F and also the energy E. From $P = -dE/dV$ get the 'degeneracy pressure' and compare it to the pressure due to gravity. They will be roughly equal. Indeed, the high degeneracy pressure is what keeps the neutron star from collapsing.