PHYSICS 200C, SPRING 2017
ELECTRICITY AND MAGNETISM

Assignment One, Due Wednesday, April 12, 5:00 pm.

[1.] In class we discussed the electric field strength in an atom, at the surface of the earth in solar radiation, and in a laser one of you used as an undergraduate. Perform a similar estimate of the electric field one centimeter away from a cell phone. One approach is to get a number for the power employed by your cell phone in broadcasting to a satellite. You could do that by looking up the energy stored in a typical cell phone battery, assuming all the energy a cell phone uses is involved in transmitting (is that right?), and then asking how long you can talk before your battery is drained.

[2.] We decided the electric field in solar radiation is "small" (in the sense that it was much, much less than the field acting on an electron due to a proton inside an atom. Does this "smallness" mean there are very few photons? Estimate the flux of visible photons a distance one meter away from a one hundred Watt light bulb. What about the number of photons per cubic wavelength 100 km away from an isotropic FM antenna with a power of 100 Watts at 10^8 Hz? Discuss the relevance of your answers to the fact that in this course we are speaking of electric and magnetic fields, and ignoring the discrete photon aspect.

[3.] A conducting metal sphere of radius a carries a free charge Q and is surrounded by a dielectric sphere of radius $b > a$. What is the potential at the center of the sphere?

[4.] Consider a point charge in a spherical tank of water. What are $\vec{E}, \vec{D}, \vec{P}$ inside the sphere? What are the induced volume and surface charge densities?

[5.] Redo problem [4] for a dipole at the center of the sphere.

Physics 200C

Assignment 1 Sol'ns

Spring 2017

$\boxed{1.}$ We'll use our more recent discussion of the Poynting vector to do this problem.

$$\text{Energy density in E field} \sim \frac{1}{2}\epsilon_0 E^2$$

Power radiated through surface of sphere of radius R $\sim c\, 4\pi R^2 \left(\frac{1}{2}\epsilon_0 E^2\right)$

charge in cell phone battery $\sim 2000\,\text{mAh}$

$$\sim 2\cdot 10^3\, 10^{-3}\, 3600 \sim 7200 \text{ Coulomb}$$

Voltage is ~ 4.2 Volts $\Rightarrow$ Energy $\sim 30000\,\text{J}$

Let's suppose all this energy is used to power the transmitter (very doubtful! what about lighting the screen and what about the efficiency of converting battery energy into EM radiation!?) Let's also suppose you can talk for 10 hours $\sim$ 36000 seconds

$$\text{Power} = 1 \text{ Watt} = \underbrace{(3\cdot 10^8)}_{c}\underbrace{4\pi(.01)^2}_{4\pi R^2}\underbrace{\frac{1}{2}(8.85\cdot 10^{-12})E^2}_{\frac{1}{2}\epsilon_0 E^2}$$

$$\Rightarrow E \sim 10^3\, N/C$$

2.

Compare this to the calculation in class of $\vec{E}$ field due to sun @ surface of earth.

$$\frac{1\,kW}{m^2} = 1000 = (3\cdot 10^8)\,\tfrac{1}{2}\,\epsilon_0 E^2$$

$\epsilon_0 = 8.85\cdot 10^{-12}$

$$\Rightarrow E \sim 10^3\ N/C$$

This seems way too large to me.

Presumably much of the power is not going to transmission of EM waves.

Can look up... Houlihan Daniel D, ANSI C63.19

$E \sim 40\ \frac{V}{m} \sim 40\ \frac{N}{C}$

So a factor of 20 lower.

2 We wish to estimate the flux of visible photons a distance 1m away from a 100W light bulb. We assume that the light bulb emits monochromatic light — photons of a single color. Let us choose $\lambda = 550\,nm$ at the center of the visible spectrum (390 - 700 nm). We have:

$$\lambda = 550\,nm \Rightarrow \nu = \frac{c}{\lambda} = \frac{3.0\times10^8}{550\times10^{-9}} = 5.45\times10^{14}\,Hz$$

The associated energy per photon is then

$$E = h\nu = (6.626\times10^{-34}\,J\cdot s)(5.45\times10^{14}\,s^{-1}) = 3.61\times10^{-19}\,J,$$

which is about 2.26 eV.

If the light bulb provides a power of $100W = 100\frac{J}{s}$, it produces

$$\frac{100\,W}{3.61\times10^{-19}\,J} = 2.77\times10^{20}\,\frac{photons}{s}$$

Consider a sphere of radius 1m. Its area is $4\pi\ m^2$. Hence, we find that the flux of photons through the surface of this sphere is

$$\text{Flux at } 1m = \frac{2.77\times10^{20}\ photons/s}{4\pi\ m^2} = \boxed{2.2\times10^{19}\,\frac{photons}{m^2 s}}$$

One could take a more sophisticated approach by considering the blackbody radiation law:

$$I(\nu, T) = \frac{2h\nu^3}{c^2}\left(e^{h\nu\beta} - 1\right)^{-1}, \text{ where } \beta = \frac{1}{kT}$$

One could then integrate this over the visible range to obtain a precise answer.

Let us now consider the number of photons per cubic wavelength 100 km away from an isotropic FM antenna with a power of 100 W at $10^8\,Hz$.

With $\nu = 10^8\,Hz$, each photon carries $E = h\nu = 6.626\times10^{-26}\,J$.

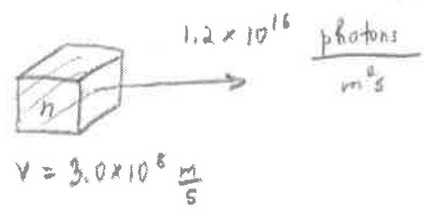
Since the radio station produces 100W, there are

$$\frac{100\,W}{6.626\times10^{-26}\,J} = 1.51\times10^{27}\,\frac{photons}{s}$$

If we consider a sphere of 100 km, this would correspond to a flux of

$$\text{Flux at } 100\,km = \frac{1.51\times10^{27}\ photons/s}{4\pi\,(10^5)^2\,m^2} = 1.2\times10^{16}\,\frac{photons}{m^2 s}$$

We first determine the number density of photons inside a box 1m on a side:



$$nv = 1.2\times10^{16}\ \frac{\text{photons}}{m^2 s}$$

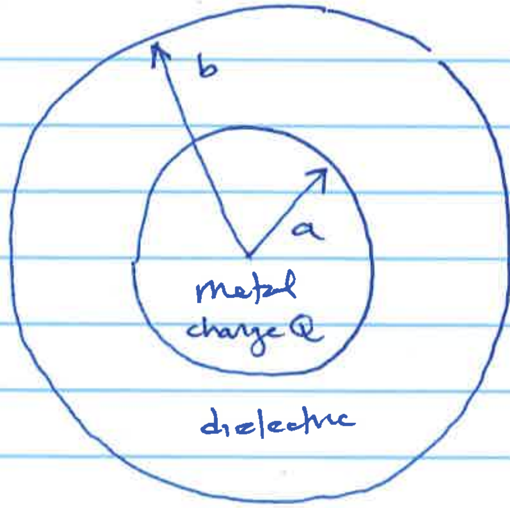
$$\Rightarrow n = \frac{1.2\times10^{16}}{3.0\times10^{8}}\ \frac{\text{photons}}{m^3} = 4.0\times10^{7}\ \frac{\text{photons}}{m^3}$$

We want to convert this to the number of photons per cubic wavelength:

$$\lambda = \frac{c}{\nu} = \frac{3\times10^{8}\ m/s}{10^{8}\ Hz} = 3\ m \Rightarrow 1\ m = \frac{\lambda}{3} \quad \text{so that} \quad 1\ m^{-3} = \left(\frac{3}{\lambda}\right)^3$$

Thus, we find $\boxed{1.1\times10^{9}\ \frac{\text{photons}}{\lambda^3}}$.

3



This is a simple gauss' law problem.

Because it is metallic, within the inner sphere $r<a$ $\vec{E}=0$

Inside the dielectric $a<r<b$ we use $\int \vec{D}\cdot\hat{n}\,dA = Q_{free}$

$$D\,4\pi r^2 = Q \qquad D = Q/4\pi r^2 \qquad E = Q/4\pi\epsilon r^2$$

reduced because of bound charge on inner surface $r=a$

Outside $r>b$ $E = Q/4\pi\epsilon_0 r^2$

To get voltage at center

$$\phi_0 = -\int_{\infty}^{b} \frac{Q}{4\pi\epsilon_0 r^2}\,dr - \int_{b}^{a} \frac{Q}{4\pi\epsilon r^2}\,dr - \int_{a} 0\,dr$$

$$= \left.\frac{Q}{4\pi\epsilon_0 r}\right|_{\infty}^{b} + \left.\frac{Q}{4\pi\epsilon r}\right|_{b}^{a}$$

$$= Q/4\pi\epsilon_0 b + Q/4\pi\epsilon a - Q/4\pi\epsilon b$$

7 Point Charge placed at the center of a Spherical Tank of Water

The geometry of the problem is shown in figure 4. We use Gauss' law to get the electric displacement in the water. The electric displacement (and electric field) is radial and independent of angle. Thus assume a small spherical shell centered on the charge. Because the filed is radial the electric displacement, D' equals the electric diaplacement in the water, D. This means;

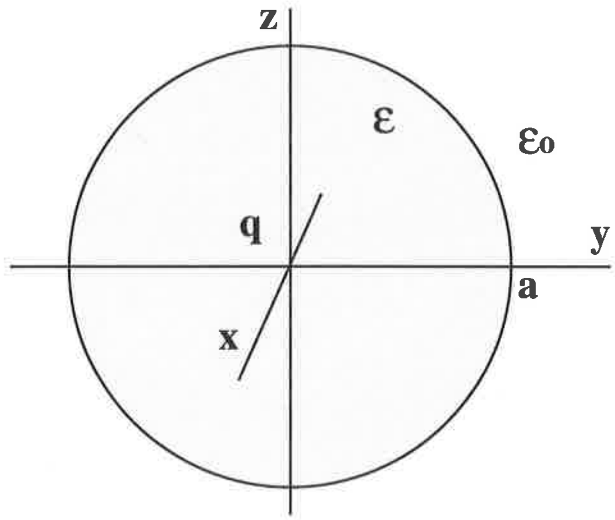


Figure 4: The geometry of a problem with a point charge q placed at the center of a spherical tank of water

$$\oint \vec{D} \cdot d\vec{A} = Q_{free} = q$$

Thus because of symmetry;

$$\vec{D} = \frac{1}{4\pi} \frac{q}{r^2} \hat{r}$$

and $\vec{D} = \epsilon \vec{E}$. Then the polarization is,

$$\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E} = \epsilon_0(\epsilon_r - 1)\frac{1}{4\pi\epsilon} \frac{q}{r^2} \hat{r}$$

The Volume charge density is ;

$$\rho = -\vec{\nabla} \cdot \vec{P} = \frac{1}{r^2} \frac{\partial}{\partial r}[r^2 P_r] = 0$$

Thus there is no volume charge density. The surface charge density at $r = a$ is;

$$\sigma = \vec{P} \cdot \hat{r} = \epsilon_0(\epsilon_r - 1)\vec{E} = \epsilon_0(\epsilon_r - 1)\frac{1}{4\pi\epsilon} \frac{q}{a^2}$$

There is an inner induced charge symmetrically placed about q at some finite radius so that the total induced charge sums to zero. Outside the water tank the field is the same as the field from a point charge q in the vacuum.

8 Dipole placed at the center of a Spherical Tank of Water

This problem is similar to the problem in the last section, but the point charge is replaced by a dipole aligned along the $\hat{z}$ axis. The field of the dipole in vacuum is;

$$\vec{E}_d = \kappa \frac{p}{r^3}[2\,cos(\theta)\hat{r} + sin(\theta)\,\hat{\theta}]$$

We put this dipole inside a small spherical volume of radius, b, in the center of the tank in order to keep the solution appropriately bounded as $r \to 0$. Thus the boundry conditions at $r = b$ are;

$$\epsilon E_r(water) = \epsilon_0 E_r(vacuum)$$

$$E_{\|}(water) = E_{\|}(vacuum)$$

Therefore inside the water;

$$\vec{E}(water) = \frac{p}{r^3}[2\epsilon_r\,cos(\theta)\,\hat{r} + sin(\theta)\,\hat{\theta}]$$

We solve for the potential in the water using separation of variables. The solution has the forms;

$$r > a$$

$$V = \sum A_l\, r^{-(l+1)}\, P_l(x)$$

$$b < r < a$$

$$V = \sum [B_l\, r^{-(l+1)} + C_l\, r^l] P_l(x)$$

Now match the boundary conditions at $r = b$.

$$\epsilon[2B_1/b^3 - C_1]cos(\theta) = 2\epsilon_0 \kappa p/b^3\, cos(\theta)$$

$$(1/b)[B_1/b^2 + C_1 b]sin(\theta) = \kappa p/b^3\, sin(\theta)$$

All other coefficients vanish. Solve for B_1 and C_1.

$$B_1 = \frac{\kappa p}{3\epsilon_r}[\epsilon_r + 2]$$

$$C_1 = \frac{2\kappa p}{3\epsilon_r b^3}[\epsilon_r - 1]$$

This gives the potential;

$$V = \frac{\kappa p}{3\epsilon_r}[\frac{\epsilon_r + 2}{r^3} + \frac{2(\epsilon_r - 1)}{b^3} r] cos(\theta)$$

From this one gets the field;

$$\vec{E} = \frac{\kappa p}{3\epsilon_r}[[\frac{\epsilon_r + 2}{r^2} + \frac{2(\epsilon_r - 1)r}{b_3}] \, cos(\theta)\,\hat{r} + \\ [\frac{\epsilon_r + 2}{r^3} + \frac{2(\epsilon_r - 1)}{b^3}] \, sin(\theta)\,\hat{\theta}]$$

The polrization is $\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$. So that the volume charge density is;

$$\rho = -\vec{\nabla}(\epsilon_0(\epsilon_r - 1)\vec{E})$$

$$\rho = \frac{\epsilon_0(\epsilon_r - 1)\,\kappa p}{3\epsilon_r r^2}[[\frac{2(\epsilon_r + 2)}{r^2} - \frac{2(\epsilon_r - 1)}{b^3}] \, cos(\theta) + \\ [\frac{\epsilon_r + 2}{r^3} + \frac{2(\epsilon_r - 1)}{b^3}] cos(\theta)]$$

$$\rho = \frac{\epsilon_0(\epsilon_r - 1)\,\kappa p}{3\epsilon_r r^2}[3(\epsilon_r + 2) + 6(\epsilon_r - 1)(r/b)^3] \, cos(\theta)$$

The surface charge density is;

$$r = b$$

$$\sigma = -(\epsilon - \epsilon_0)\frac{8\kappa p}{3\epsilon_r b^3} \, cos(\theta)$$

$$r = a$$

$$\sigma = -(\epsilon - \epsilon_0)\frac{4\kappa p}{3\epsilon_r a^3}[\epsilon_r(1 - \epsilon_r(a/b)^3) + 2(a/b)^3] \, cos(\theta)$$

Matching the boundry conditions at $r = a$ must now be carefully done. As the field does not $\rightarrow 0$ as $r \rightarrow \infty$ but has a dipole form, with the potential given by;

$$V = \frac{2\kappa p(\epsilon_r - 1)}{b^3} \, r \, cos(\theta)$$

This potential should be subtracted from the dipole potential. The problem comes from defining a dipole as a point.