# PHYSICS 200B, WINTER 2017
# ELECTRICITY AND MAGNETISM

## MIDTERM EXAM

**[1.]** Compute the potential $V(r, \theta, \phi)$ due to a thin ring of charge $Q$ and radius $R$. Make a convenient choice of origin and orientation of your axes, and assume $r > R$. Interpret the term which falls off most slowly with $1/r$. How does your calculation change for $r < R$?

**[2.]** A crude model of the $H_2$ molecule is that the electrons form a spherical cloud of charge of radius $a$ and the two protons are point charges inside this sphere. Find the equilibrium proton positions.

**[3.]** One can verify by explicit integration that the functions $\sin(n\pi x/L)$ and $\cos(n\pi x/L)$, where $n = 1, 2, 3, \cdots$, are orthogonal and complete on the interval $x \in [0, L]$. This is the basis of Fourier expansion. Similarly, given the specific functional forms of the Legendre polynomials, they can be shown to also to be orthogonal and complete. Write a few sentences describing what more general principle might lie behind the idea of complete sets of functions. Can you make and analogy with the eigenvectors of a specific class of matrices?

**[4.]** Use the generating function for the Legendre polynomials

$$g(x, t) = \frac{1}{\sqrt{1 - 2xt + t^2}} = \sum_n t^n P_n(x)$$

to prove $P_n(1) = 1$ for all $n$. What can you say about $P_n(0)$?

*Do only one of problems [5] or [6] below. In either case, solve the problem completely from scratch, ie starting from the appropriate partial differential equation, making a suitable guess at the form of the solution, etc.*

**[5.]** Solve for the potential $V(x, y)$ inside a rectangular box of dimensions $0 < x < b$ and $0 < y < h$ given the boundary conditions $V(x, y = 0) = 0$; $V(x = 0, y) = 0$; $V(x = b, y) = 0$ and $V(x, y = h) = f(x)$ where $f(x)$ is an arbitrary function which vanishes at $x = 0$ and $x = b$. There is no charge inside the box. Identify the Green's function which arises in your solution.

**[6.]** Solve for the potential $V(x, y)$ in the upper half-plane $y > 0$ if you are given the potential $V(x, y = 0) = f(x)$ along the $x$ axis. There are no charges present. Suppose $f(x) = V_0$ is constant. What do you get for the potential $V(x, y)$? Identify the Green's function which arises in your solution.

**Potentially Useful Identity:**

$$(1 + u)^n = 1 + n\,u + \frac{n(n-1)}{2}\,u^2 + \frac{n(n-1)(n-2)}{6}\,u^3 \cdots$$