QMGTI

time independent

The schroedinger eqn

$$\hat{H}\psi = \left[P^2/2m + V\right]\psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\psi$$

is invariant under global changes of phase of ψ

$$\psi \to \psi' \equiv e^{i\Lambda}\psi \qquad \Lambda = \text{indep of } x$$

Notice $p(\vec{x}) = |\psi(\vec{x})|^2$ is also unchanged.

This invariance of $p(\vec{x})$ also is true under local changes

$$\psi \to \psi' \equiv e^{i\Lambda(\vec{x})}\psi$$

It is more convenient to write this as (more later on this...)

$$\psi \to \psi' \equiv e^{i e/\hbar\, \Lambda(\vec{x})}\psi$$

Now the Sch Eqn is invariant if we replace

$$\hat{H} = \left[\frac{(\vec{P} - e\hat{A})^2}{2m} + V\right]$$

and require $\vec{A} \to \vec{A} + \nabla\Lambda$ when we change $\psi \to e^{i\Lambda}\psi$

QMGT2

The proof is simple

$$(p - eA')\psi' = \left[\frac{\hbar}{i}\nabla - e(A + \nabla\Lambda)\right] e^{ie\Lambda/\hbar}\psi$$

$$= e^{ie\Lambda/\hbar}\frac{\hbar}{i}\nabla\psi + e\nabla\Lambda\, e^{ie\Lambda/\hbar}\psi - eAe^{ie\Lambda/\hbar}\psi - e\nabla\Lambda e^{ie\Lambda/\hbar}\psi$$

cancel

$$= e^{ie\Lambda/\hbar}(p - eA)\psi$$

This appears also in $V\psi$ and $E\psi$ terms and hence cancels throughout.

The choice $e^{ie\Lambda/\hbar}$ is also motivated dimensionally

$$[B] = \frac{\text{FORCE}}{Q\ \text{VELOCITY}} = \frac{ML}{QT^2}\frac{T}{L} = \frac{M}{TQ}$$

since $B = \nabla\times A$: $[A] = [B]L = ML/TQ$

since $A \to A + \nabla\Lambda$ $\quad [\Lambda] = [A]L = [B]L^2 = ML^2/TQ = [\Phi_B]$

Finally $\quad [e/\hbar] = Q/ML^2/T = QT/ML^2 = 1/[\Phi_B]$

so $\quad e\Lambda/\hbar$ is dimensionless.

U1

$\boxed{\text{SI Units for } E, B}$

$$[E] = \frac{\text{Force}}{Q} = \frac{ML}{T^2Q}$$

From $\vec{E} = -\vec{\nabla}V$

$$[V] = [E]\cdot L = ML^2/T^2Q$$

check $[V]Q = [\text{Energy}] = \frac{ML^2}{T^2}$ ✓✓

$$[B] = \frac{\text{Force}}{QL/T} = \frac{ML}{T^2Q}\frac{T}{L} = \frac{M}{TQ}$$

From $\vec{B} = \vec{\nabla}\times\vec{A}$

$$[A] = [B]\cdot L = ML/TQ$$

From $A \to A + \nabla\Lambda$

$$[\Lambda] = [A]L = ML^2/TQ$$

So a dimensionless $[\Lambda]$ would be $\frac{TQ}{ML^2}[\Lambda]$

$$[\phi_B] = [B](\text{area}) = \frac{ML^2}{TQ} = [\Lambda]$$

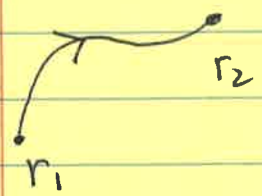
SI flux quantum is $\frac{h}{2e}$

$$\frac{[h]}{[e]} = \frac{[ML^2/T^2\,T]}{Q} \quad ✓✓$$

QM 4 T 2'

Put another way, when a QM particle traverses a region of nonzero $\vec{A}$ it picks up a phase factor due to $\vec{A}$

$$\Delta\phi = \frac{e}{\hbar}\int_{r_1}^{r_2} \vec{A}\cdot d\vec{r}$$



This phase factor depends on choice of gauge, however going around a closed loop does not depend on choice of gauge

$$\oint \vec{\nabla}\Lambda\cdot d\vec{r} = \int \underbrace{(\nabla\times\nabla\Lambda)}_{\emptyset}\cdot\hat{n}\,dA$$

$$\frac{\hbar}{i}\nabla\psi = E\psi$$

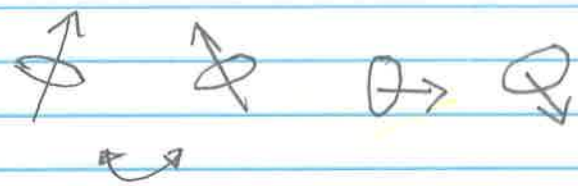
$$\psi = e^{i\vec{k}\cdot\vec{r}} \qquad E = \hbar^2k^2/2m$$

as particle moves its phase changes from $\vec{k}\cdot\vec{r}_1$ to $\vec{k}\cdot\vec{r}_2$

from $\vec{r}_1$ to $\vec{r}_2$

GSMO

One of most interesting aspects of magnetism: "magnetic order". Today's story is about role of gauge degrees of freedom in magnetic order.



magnetic moments in solids eg spins of e^- or orbital moments of e^-

interaction Energy

Thermal fluctuations
+
quantum fluctuations

$[\hat{p},\hat{x}] \neq 0$ — just as you have uncertainty principle in particle positions velocity

$[\hat{S}_x,\hat{S}_y] \neq 0$ — so too in the orientation of their magnetic moment

You studied $E = -J \sum \cos(\theta_i - \theta_j)$

a classical model of magnetism (no quantum fluctuations)

$\theta_i = \theta_0 \ \forall i$ minimizes $\langle E \rangle = -NJ$

GSM1

The discussion of gauge degrees of freedom in EM might leave the impression that the presence of a freedom in the choice of potential does not affect the physics. You choose a gauge but whichever you choose to calculate in, the physics is the same. This is true

However, the extra degrees of freedom can have an effect at finite temperature, since they increase the possible configurations accessible to the system, hence the entropy, and decrease the likelihood of order.

Consider the Ising model $S_i = \pm 1$ on each site i of a lattice

$$E = -J \sum_{\underbrace{\langle ij \rangle}_{\text{neighbors}}} S_i S_j$$

neighbors	Energy	
$++$	$-J$	Ferromagnetic
$--$	$-J$	
$+-$	$+J$	
$-+$	$+J$	

Two ground states

$$\begin{array}{cccc} + & + & + & + \\ + & + & + & + \\ + & + & + & + \\ + & + & + & + \end{array} \qquad \begin{array}{cccc} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{array}$$

$$E_0 = - J \underbrace{N}_{\#\text{ sites}} q/2$$

$q = \#$ neighbors
(coordination $\#$)

Nature minimizes Free energy

$$F = E - TS$$

GSM-2

Many more (exponentially more!) high energy states

+ − − +
− + + − 2^N total states
− − + −
+ − − +

$F = E - TS$ ← maximize

F ← minimize; E ← minimize

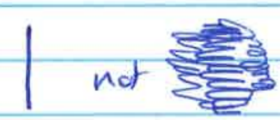
minimize ⇒

"T" is knob controlling relative weight of E vs S in minimizing F

minimize E ⇒ ordered phase

minimize F ⇒ disordered phase

Surprise is that transition is sharp!



Aside: Ising Model transfer matrix in one dimension

partition function $Z = \sum_{\{S_\ell\}} e^{-\beta E} = \sum_{\{S_\ell\}} e^{\beta J \sum_i S_i S_{i+1}}$

$$M = \begin{array}{c|cc} S_1 \backslash S_2 & 1 & -1 \\ \hline 1 & e^{\beta J} & e^{-\beta J} \\ -1 & e^{-\beta J} & e^{\beta J} \end{array}$$

$$= \sum_{S_1} \sum_{S_2} .. \sum_{S_N} \underbrace{e^{\beta J S_1 S_2}}_{M(S_1 S_2)} \underbrace{e^{\beta J S_2 S_3}}_{M(S_2 S_3)} \cdots \underbrace{e^{\beta J S_N S_1}}_{M(S_N S_1)}$$

$$= \sum_{S_1} \sum_{S_3} \cdots \sum_{S_N} M^2(S_1 S_3) \cdots M(S_N S_1)$$

$$= \sum_{S_1} M^N(S_1 S_1) = \mathrm{Tr}\, M^N = (2\cosh\beta J)^N + (2\sinh\beta J)^N$$

Eigenvalues of M are $\lambda_{1,2} = 2\cosh\beta J\ ;\ 2\sinh\beta J$

GSM 3

Ising model has global spin flip symmetry

$S_i = -S_i$ on all i simultaneously : E is unchanged

$$E = -J \sum_{\langle ij \rangle} S_i S_j$$

like
$\psi(x) \to e^{i\Lambda} \psi(x)$
at all x simultaneously

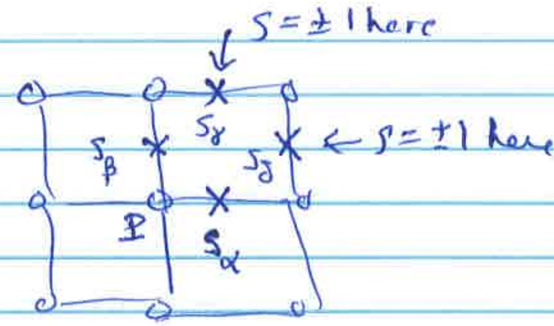
$$\to -J \sum_{\langle ij \rangle} (-S_i)(-S_j)$$

global discrete symmetry Z_2
XY model has global $U(1)$ symmetry

Conventional Ising Model $S_i = \pm 1$ live on sites of lattice

E involves $S_i S_j$ (ij) neighbors on bond

Wegner model $S = \pm 1$ live on bonds of lattice



$$E = -J \sum_{\square} S_\alpha S_\beta S_\gamma S_\delta$$

around all "plaquettes" of lattice.

This model has a local spin flip symmetry:

Pick a lattice site P and $S_\alpha \to -S_\alpha$ for all

spins on bonds emanating from P

GSM 4

eg # of ground states

The additional freedom for local gauge transformation

inhibits phase transitions. Conventional Ising has phase transition

in $d=2$, Wegner does not (requires $d=3$).

What can you meaningfully measure in Wegner?

product of S_α around closed loop (arbitrary size)

is "gauge invariant"

All this has close analogs in quantum field theory

$\left\{\begin{matrix} E, B \\ A, \phi \end{matrix}\right\}$	mediated electric forces QED	charged particles (electrons)
$\left\{\begin{matrix} \text{gluon} \\ \text{fields} \end{matrix}\right\}$	mediate strong forces	strongly interacting particles (quarks)

closed loops in Wegner $\leftrightarrow$ confinement in QCD