

**PHYSICS 200B, WINTER 2017**  
**ELECTRICITY AND MAGNETISM**

**Assignment Six, Due Monday, March 20, 5:00 pm.**

[1.] The following three vector potentials describe the same magnetic field:

$$B_0 x \hat{\mathbf{y}}, \quad -B_0 y \hat{\mathbf{x}}, \quad \frac{1}{2} B_0 (x \hat{\mathbf{y}} - y \hat{\mathbf{x}}).$$

Find the magnetic field, and the gauge transformations relating the different choices of  $\mathbf{A}$ .

[2.] Consider the vector potential  $\mathbf{A}(\mathbf{r}) = A_0 e^{-(x^2+y^2+z^2)/a^2} \hat{\mathbf{z}}$ , where  $A_0$  and  $a$  are constants.

(a) Find and sketch the corresponding magnetic field.

(b) Can this be a *magnetostatic* field? If yes, find the current distribution that would give rise to it, and if not, explain why not.

(c) Is the vector potential given in the Coulomb gauge? If not, transform it to the Coulomb gauge to the best of your ability.

[3.] There is an interstellar magnetic field throughout the Milky Way with a strength between 1 and  $10 \mu\text{G}$ . Taking an intermediate value of  $3 \times 10^{-6} \text{ G}$ , and modelling the galaxy as a disk of diameter  $\sim 10^{23} \text{ cm}$ , and thickness  $10^{21} \text{ cm}$ , find the magnetic energy stored in the galaxy in order of magnitude. To get a sense of whether this is big or small, consider that all the stars in the galaxy put together are radiating  $10^{44} \text{ ergs/sec}$ . How many years of starlight is the magnetic energy worth?