PHYSICS 200B, WINTER 2017
ELECTRICITY AND MAGNETISM

Assignment Five Due Monday, March 13, 5:00 pm.

[1.] Verify that $(\mu_0 \,/\, 4\pi)\,\mathbf{m}\times\mathbf{r}/r^3$ is a suitable vector potential for a point magnetic dipole, i.e. that its curl gives a dipole magnetic field. Assume $r \neq 0$.

[2.] Find the vector potential and magnetic field of an infinite straight wire of cross section with radius a, carrying a current I uniformly distributed across its area. (A good starting point is the thin wire $a \to 0$ done in class.)

[3.] When a type-II superconductor is placed in a magnetic field, the field becomes non-uniform and passes through the superconductor in thin cylindrical regions called *flux tubes* or *vortices*. (See figure at left below.) Here we study a toy model of a flux tube. Take $\mathbf{B}$ parallel to $\hat{\mathbf{z}}$ everywhere, the z axis along the axis of the tube, and $r_\perp = (x^2+y^2)^{1/2}$. Assume the magnetic flux through a circle of radius $r_\perp$ centered on the axis of the tube is

$$\Phi(r_\perp) = 2\Phi_0 \frac{r_\perp^2}{r_\perp^2 + a^2} \qquad\qquad (\Phi_0, a \text{ are constants}).$$

Find $\mathbf{A}(\mathbf{r})$, $\mathbf{B}(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$ accompanying the tube. Make informatively marked and labelled sketches showing the spatial variation of $\mathbf{B}, \mathbf{j}$ and Φ.

[4.] A circular wire loop of radius R_1 carrying current I_1 is placed in the xy plane with its center at the origin. A second circular wire loop of radius R_2, carrying current I_2 lies parallel to the first one, with its center at $(0, 0, h)$. Assume the first loop is very small so that $R_1 \ll h$ and $R_1 \ll R_2$. See figure at right below.

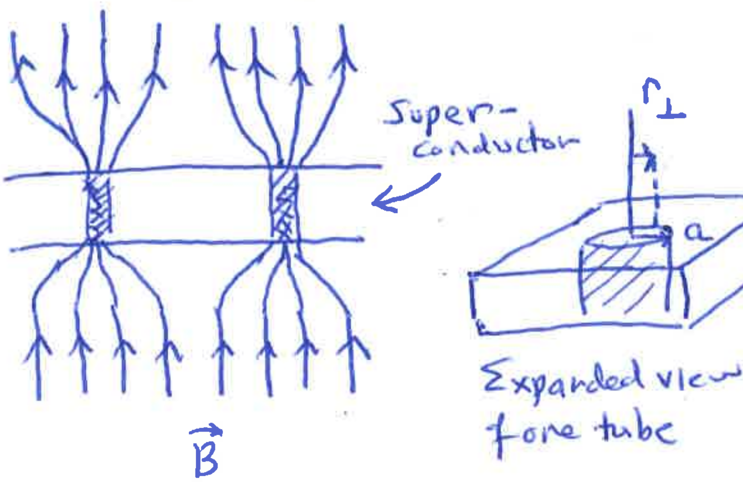
Show that the field due to the upper loop is given by

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R_2^2 I_2}{R_2^2 + (z-h)^2}$$

at points along the z axis. Using the fact that R_1 is very small, compute the magnetic flux Φ_{12} through the small loop due to the field created by the big loop. Find the mutual inductance $L_{12} = \Phi_{12}/I_2$.

Using a physically equivalent set of approximations, find the magnetic flux Φ_{21} through the big loop due to the field created by the small loop. Compute L_{21} and show $L_{21} = L_{12}$. (*Do not use this theorem to do the problem!*)

PROBLEM 3:



PROBLEM 4: