PHYSICS 200B, WINTER 2017
ELECTRICITY AND MAGNETISM

Assignment One, Due Friday, January 20, 5:00 pm.

[1.] A crude model of the H_2 molecule is that the electrons form a spherical cloud of charge of radius a and the two protons are point charges inside this sphere. Find the equilibrium proton positions.

[2.] In certain liquids, when a point impurity q is placed at the origin, the electrostatic potential is,

$$\phi(r) = \frac{q}{4\pi\epsilon_0 r}\, e^{-r/\lambda} \ .$$

λ is called the Debye-Huckel length. Find the electric field and charge density everywhere, and the total charge inside an infinitely large sphere.

[3.] A hydrogen atom acts as if it had an electrostatic potential

$$\phi(r) = \frac{q}{4\pi\epsilon_0 r}(1 + \frac{r}{a_0})e^{-2r/a_0} \ ,$$

where q is the charge on the proton and $a_0 = \hbar^2/m_e q^2 = 0.529\dot{A}$ is the Bohr radius.
(a) Find the corresponding charge density and interpret the various terms physically.
(b) Verify by *explicit* integration that your resulting charge density from part (a) indeed produces the original potential.
(c) What is the net charge inside a sphere of radius a_0? What is the electric field strength at this distance?

[4.] A sphere of radius a has a charge Q uniformly distributed thhroughout its volume.
(a) Obtain the total electrostatic energy $3Q^2/(20\pi\epsilon_0 a)$ directly from the expression,

$$U = \frac{1}{8\pi\epsilon_0}\int d^3r \int d^3r' \ \frac{\rho(\vec{r})\rho(\vec{r}\,')}{|\vec{r} - \vec{r}'|} \ .$$

(b) Use Gauss' Law to find the electric field throughout all space. Rederive the total energy by integrating $\frac{1}{2}\epsilon_0 E^2$.
(c) Suppose the same charge is instead spread uniformly over the *surface* of the sphere. Use either of the above methods to find the corresponding total energy, and discuss *physically* why and how it differs from the uniformly charged sphere.

[5.]
(a) Suppose $G(\vec{r}, \vec{r}')$ satisfies the inhomogeneous equation $\nabla_r^2 G(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}')/\epsilon_0$, subject to the boundary condition $G(\vec{r}, \vec{r}') = 0$ for $\vec{r}$ on all surrounding surfaces $\mathcal{S}$. Use Green's theorem to solve the equation $\nabla_r^2 \phi(\vec{r}) = -\rho(\vec{r})/\epsilon_0$, with the specified boundary conditions $\phi(\vec{r}) = -f(\vec{r})$ on $\mathcal{S}$.
(b) As an example, consider the empty half-space $z > 0$. The boundary plane at $z = 0$ is held at zero potential except for an equilateral triangle of side a at a potential V. Find the appropriate $G(\vec{r}, \vec{r}')$ and hence obtain an integral expression for the potential throughout the region $z > 0$. Evaluate the first nonvanishing term in $\phi(\vec{r})$ as $|\vec{r}| \to \infty$.

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[5.] (continued)
(c) A point charge q is now added to the configuration in part (b). The charge is placed a distance z_0 from the plane along a line through the center of the triangle and normal to its plane. Find an integral representation for the total potential and evaluate the first nonvanishing contribution for $|\vec{r}| \to \infty$. Identify and interpret the various terms.

[6.] A conducting sphere of radius a has a total charge Q. Of this charge, $Q - Q_E$ is distributed uniformly through the volume of the sphere, while the remaining Q_E is uniformly distributed in a ring of zero thickness around the equator of the surface of the sphere. Assume that Q and Q_E are of the same sign.
(a) By direct integration, find the potential $\phi(\vec{r})$ along the polar axis of the sphere for $|\vec{r}| > a$.
(b) By expressing the potential in a power series in spherical coordinates, and comparing with your answer in (a), determine $\phi(\vec{r})$ for all points $\vec{r}$ such that $|\vec{r}| > a$.
(c) Calculate the electric field for large $|\vec{r}|$ to order $a^2/|\vec{r}|^2$. Then consider a particle of charge q (q and Q are of opposite signs) which is held by electrostatic attraction in a bound orbit around the sphere. Show that such an orbit is stable in *inclination* (the angle between the orbit plane and the equatorial plane of the sphere) only if the inclination is zero. Discuss the gravitational analog of this problem. (Recall the earth has an equatorial bulge.)