SQ 1

I have tried to motivate fermion creation / destruction operators by analogy with s.h.o. Perhaps that was not so useful

Now I will proceed instead "postulationally"

Traditional __First__ __quantized__ QM

1) State of system represented by vector $|\psi(t)\rangle$
2) Observables by Hermitian operators / matrices A
3) Eigenvalues of operator $\leftrightarrow$ possible results of expt

$$A|\phi_n\rangle = a_n|\phi_n\rangle$$

prob of measuring a_n is $|\langle\phi_n|\psi\rangle|^2$ $\qquad |\psi\rangle = \sum_n c_n|\phi_n\rangle$

$$c_n = \langle\phi_n|\psi\rangle$$

4) $[x,p] = i\hbar$
5) $|\psi(t)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle$

Then did many examples

SQ3

$$c_4^\dagger c_2^\dagger |VAC\rangle = -c_4^\dagger c_2^\dagger |VAC\rangle \quad !$$

$\therefore$ Need convention, eg. $|0010101 0 0100\rangle$

$$\equiv c_8^\dagger c_5^\dagger c_3^\dagger |VAC\rangle$$

lowest index at right

+(ii)' Occupation # states are basis

$$|\psi(t)\rangle = \sum_{\{n_\ell\}} (\text{Coefficients}) \, |n_1 n_2 \ldots n_N\rangle$$

N.B. issue of basis arises in first quantized QM as well

$$\hat{X}|x\rangle = x|x\rangle \qquad \psi(x) = \langle x|\psi\rangle$$

$$\hat{P}|p\rangle = p|p\rangle \qquad \phi(p) = \langle p|\psi\rangle$$

related by FT

$$|\psi\rangle = \int dx \, \langle x|\psi\rangle \, |x\rangle$$

$$|\psi\rangle = \int dp \, \langle p|\psi\rangle \, |p\rangle$$

like our occ # basis

coefficients

SQ 4

4) LAST STEP JUST like first quantized QM

Hamiltonian H

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$

How did you usually do first quantized QM?

$$\hat{H}|\phi_n\rangle = E|\phi_n\rangle$$

$$|\psi(0)\rangle = \sum_n c_n|\phi_n\rangle$$

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar}|\phi_n\rangle$$

It all started with choosing $\hat{H}$ of interest

and a basis and calculating eigenfunctions/eigenvalues.

Often useful to identify other operators

commuting with $\hat{H}$ eg L_z, L^2 to help in search.

SQ3'

The number operator

$$c_2^\dagger c_2 |1010\rangle$$

$$= c_2^\dagger c_2 c_3^\dagger c_1^\dagger |0000\rangle$$

$$= - c_2^\dagger c_3^\dagger c_2 c_1^\dagger |0000\rangle$$

$$= + c_2^\dagger c_3^\dagger c_1^\dagger c_2 |0000\rangle = 0$$

$$c_3^\dagger c_3 |1010\rangle$$

$$= c_3^\dagger c_3 c_3^\dagger c_1^\dagger |0000\rangle$$

$$= c_3^\dagger (1 - c_3^\dagger c_3) c_1^\dagger |0000\rangle$$

$$= c_3^\dagger c_1^\dagger |0000\rangle - \underbrace{c_3^\dagger c_3^\dagger}_{\emptyset} c_3 c_1^\dagger |0000\rangle$$

$$= 1 |1010\rangle$$

$$c_e^\dagger c_e |n_1 n_2 \ldots n_N\rangle = n_e |n_1 n_2 \ldots n_N\rangle$$

SQ 3''

Another example of manipulation

$c_2^+ c_3 |1\,0\,1\,0\rangle$

$= c_2^+ c_3\ c_3^+ c_1^+ |0\,0\,0\,0\rangle$

$= c_2^+ (1 - c_3^+ c_3) c_1^+ |0\,0\,0\,0\rangle$

$= c_2^+ c_1^+ |0\,0\,0\,0\rangle - c_2^+ c_3^+ c_3 c_1^+ |0\,0\,0\,0\rangle$

$\vdots$

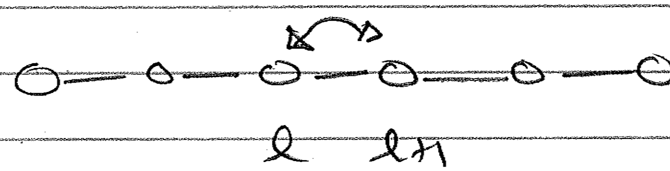
$= |1\,1\,0\,0\rangle$

SQ5

Let's do that here

$$\hat{H} = -t \sum \left(c_\ell^\dagger c_{\ell+1} + c_{\ell+1}^\dagger c_\ell \right)$$

Why: this is like free particle KE



$[\hat{H}, \hat{N}] = 0$

"Second Quantized" QM $\left\{ \begin{array}{l} \text{on a lattice CM} \\ \text{continuum HET (except LGT)} \end{array} \right.$

(1) state of system represented by linear combination of "occupation # states"

$|n_1 n_2 n_3 \ldots n_N\rangle$ $n_\ell = 0,1 = \#$ electrons on site ℓ (nucleus ℓ)

$|VAC\rangle = |000 \ldots 0\rangle$

(2) creation destruction operators act on occ # state eg

$c_\ell^\dagger |000 \ldots 0\rangle = |000 \ldots 0\underset{\ell\text{th spot}}{1}0 \ldots 0\rangle$

↑ VAC

$c_\ell |000 \ldots 0\rangle = 0$ ← or any time ℓth site empty

(3) $\{c_\ell, c_j\} = \{c_\ell^\dagger, c_j^\dagger\} = 0$ ← PAULI PRINCIPLE and ANTISYMMETRY

$\{c_\ell, c_j^\dagger\} = \delta_{\ell j}$

$c_\ell^\dagger |000 \ldots 0\underset{\ell}{1}0 \ldots 0\rangle$

$= c_\ell^\dagger c_\ell^\dagger |VAC\rangle = \phi$

Recall ASYM $\psi(r_1, r_2) = \pm \psi(r_2, r_1)$

BOSONS / FERMIONS

$\psi(r_1 r_2 \ldots r_\ell \ldots r_j \ldots r_N)$

$= -\psi(r_1 r_2 \ldots r_j \ldots r_\ell \ldots r_N)$