FSW-Ø

Why quantization?

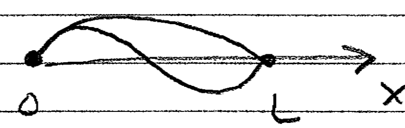
Vibrating string

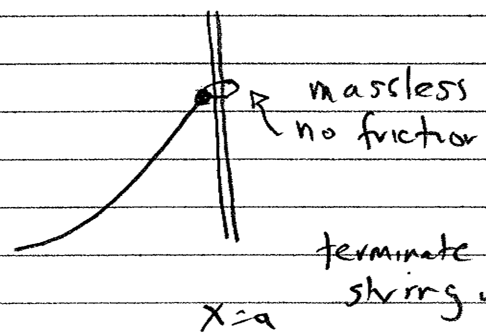
wave eqn $\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$

solns $\sin(kx - kvt) = y(x,t)$ **any** k

But Boundary conditions $y(x=0,t) = y(x=L,t) = 0$

forces **quantized** $k = n\pi/L$

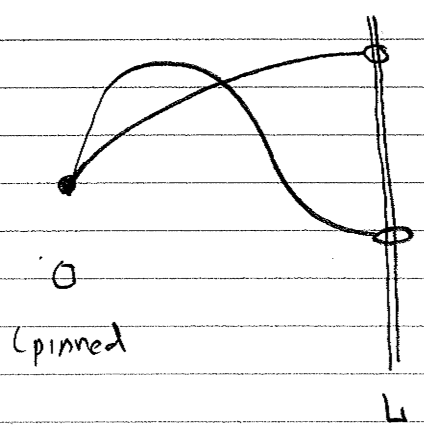




terminate string with ring

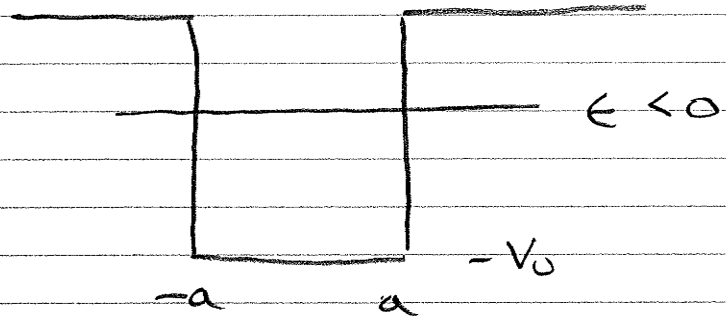
Bdy condition

$\left.\partial y/\partial x\right|_{x=a} = 0$



$k = \frac{\pi}{2L}, \frac{3\pi}{2L}, \ldots$

Finite Square well



$\epsilon > -V_0$

$\epsilon + V_0 > 0$

Outside well $\quad -\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} = \epsilon\psi$

$\uparrow$ $\epsilon < 0$

$x > 0$ $\quad \to \psi(x) = e^{Qx},\ e^{-Qx}$

$\psi(x) = Ae^{-Qx}$

$$\frac{\hbar^2 Q^2}{2m} = -\epsilon$$

Inside well $\quad -\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} - V_0\psi = \epsilon\psi$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = \underbrace{(V_0 + \epsilon)}_{>0}\psi \to \psi(x) = \sin kx \ \cos kx$$

$$\frac{\hbar^2 k^2}{2m} = (V_0 + \epsilon)$$

Inside well wave like vibrating string

But at well wall wave has specified slope, must

match up $\quad \dfrac{d\psi}{dx} = \dfrac{d}{dx} Ae^{-Qx} = -QAe^{-Qa}$

This Known slope forces k to be quantized (just like vibrating string!)

FSW - 2

Look for even sol'ns (since $V(x) = V(-x)$ sol'ns will be even or odd)

$P f(x) = f(-x)$ parity operator

$HP = PH$ $\qquad P^2 = I$

$Pv = \lambda v$

$P^2 v = \lambda^2 v = v \qquad \lambda = \pm 1$ even / odd

$$\psi(x) = \begin{cases} Ae^{-Qx} & x > a \\ B\cos kx & x < a \end{cases}$$

$Ae^{-Qa} = B\cos ka$

$-QAe^{-Qa} = -kB\sin ka$

Dividing $\quad k\tan ka = Q$

Define $\quad z = ka \quad z_0 = \frac{a}{\hbar}\sqrt{2mV_0}$

$$\frac{\hbar^2 k^2}{2m} = (V_0 + \epsilon) = V_0 - \frac{\hbar^2 Q^2}{2m} \Rightarrow \frac{\hbar^2}{2m}(Q^2 + k^2) = V_0$$

$$\frac{\hbar^2 Q^2}{2m} = -\epsilon \qquad Q^2 + k^2 = \frac{2mV_0}{\hbar^2} = \left(\frac{z_0}{a}\right)^2$$

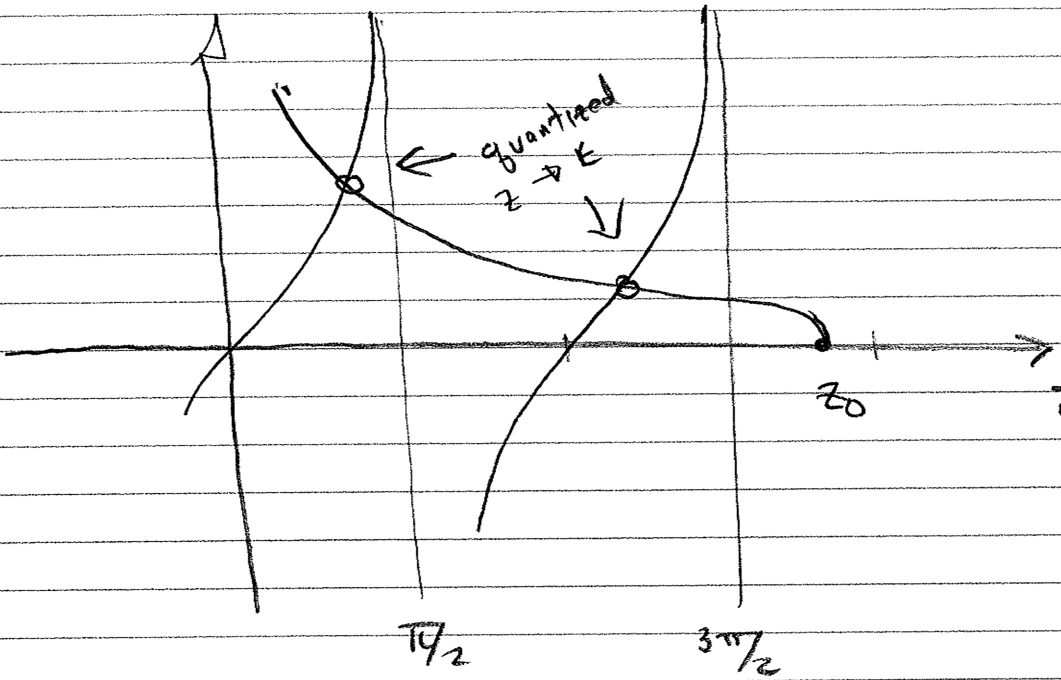
$$Q^2 a^2 = z_0^2 - k^2 a^2 = z_0^2 - z^2$$

$$\tan ka = \frac{Q}{k} = \frac{Qa}{ka} = \frac{\sqrt{z_0^2 - z^2}}{z} = \sqrt{\left(\frac{z_0}{z}\right)^2 - 1}$$

$\tan z$

FSW-3

$$\tan z = \sqrt{\left(\frac{z_0}{z}\right)^2 - 1} \qquad z_0 = \frac{a}{\hbar}\sqrt{2mV_0}$$



$\sqrt{\left(\frac{z_0}{z}\right)^2 - 1}$ ends when

$$\frac{z_0}{z} = 1$$

$$z = z_0$$