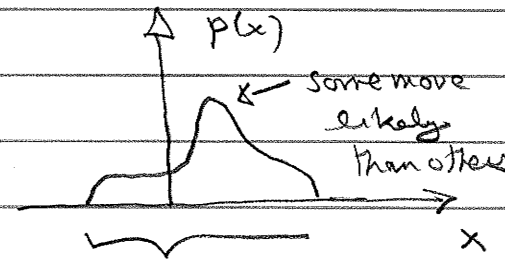
DOS-1

Density of States

Energy band ← continuous set of energy levels.

But not all energies in band are equally likely.

analogy: probability distribution



range of allowed x values

$p(x)\,dx$ = probability measure $(x, x+dx)$

For example for free particles in 3D

$$E = \frac{\hbar^2 k^2}{2m}$$

$$N = 2 \frac{V}{(2\pi)^3} \int d^3k = 2 \frac{V}{(2\pi)^3} \int_0^{k_F} 4\pi k^2\, dk$$

spin

$\left(\frac{2\pi}{L}\right)^3$ volume per k point

$$= \frac{V}{\pi^2} \frac{k_F^3}{3}$$

$$k_F = (3\pi^2 \rho)^{1/3}$$

$n(k)\,dk \equiv$ # $\vec{k}$ vectors with $|\vec{k}|$ between k and $k+dk$

$$= \underbrace{\frac{V}{(2\pi)^3} 4\pi k^2}_{n(k)}\, dk = n(E)\, dE$$

$$E = \frac{\hbar^2 k^2}{2m} \qquad dE = \frac{\hbar^2 k}{m}\, dk$$

$$n(k)\,dk = \frac{V}{(2\pi)^3}\, 4\pi k^2 \frac{m\,dE}{\hbar^2 k}$$

$$= \underbrace{\frac{V}{2\pi^2} \frac{m}{\hbar^2} \left(\frac{2mE}{\hbar^2}\right)^{1/2}}_{n(E)} dE$$

(Sometimes add factor of 2 for spin.)

$N(E) \sim E^{1/2}$ for free fermions

↑ proportionality constant involves mass

Why impt? Recall $C \sim \gamma T$ for Fermions

because only states within $k_B T$ of Fermi surface

can respond to increase in temperature.

$$C = \frac{3}{2} N k_B \quad \text{classical}$$

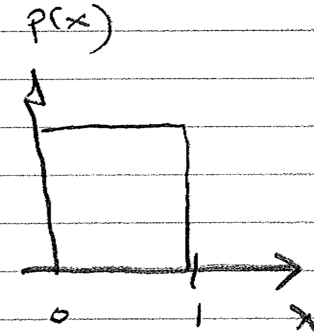
$$\downarrow$$

$$N \frac{k_B T}{E_F} k_B$$

Exact result is $\frac{\pi^2}{2} \frac{k_B T}{E_F} N k_B$

DOS-1'

Mathematical Aside



Suppose X is uniform on $[0,1]$

ie $p(x) = 1 \quad 0 < x < 1$

You use these x values to generate $y = x^2$.

what is $\tilde{p}(y)$? Is it uniform?

$x=$	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
$y=$	.01	.04	.09	.16	.25	.36	.49	.64	.81	1.0

$\frac{3}{10}$ values interval $0 < y < .1$

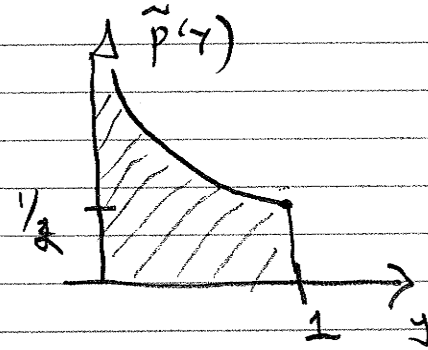
Seems like y small more likely

Method to get $\tilde{p}(y)$ is very simple

$$\tilde{p}(y)\,dy = p(x)\,dx \qquad y = x^2 \quad dy = 2x\,dx$$

$$\tilde{p}(y)\,dy = 1 \quad dy/2\sqrt{y}$$

$$\therefore \tilde{p}(y) = \frac{1}{2\sqrt{y}}$$



NB $\int_0^1 \tilde{p}(y)\,dy = 1 \checkmark$

It's not completely obvious $p(y)dy = p(x)dx$

Here's another argument:

$\tilde{p}(y)dy \equiv$ prob get value between y and $y+dy$

For this to happen x must be between $\sqrt{y}$ and $\sqrt{y+dy}$

But $\sqrt{y+dy} = \sqrt{y(1+dy/y)} = \sqrt{y}\,(1+dy/y)^{1/2}$

$$\doteq \sqrt{y}\left(1+\frac{dy}{2y}\right) = \sqrt{y} + \frac{dy}{2\sqrt{y}}$$

The likelyhood x is in any range x_1, x_2 is $x_2 - x_1$

so the probability is just $\dfrac{dy}{2\sqrt{y}} = \tilde{p}(y)dy$

This yields same answer $\tilde{p}(y) = \dfrac{1}{2\sqrt{y}}$

Bottom line if you have x with $p(x)$

and $y = f(x)$ and you want $\tilde{p}(y)$

Just set $p(x)\,dx = \tilde{p}(y)dy$

$dy = f'(x)dx$ eliminate all references to x.

States within $k_B T$ of E_F involves $N(E_F)$

$C \sim N(E_F)$

* Many responses of solid involve $N(E_F)$ and

indeed become larger as $N(E_F)$ increases (more e^-

are free to participate) $\chi = dM/dB \sim N(E_F)$

magnetic susceptibility

Superconductivity

$$T_c \sim \omega_{phonon}\, e^{-1/VN(E_F)}$$

electron phonon interaction energy

"Heavy" fermions Curro, Zieve:

$CeAl_2$, $CeAl_3$, $CeCoIn_5$, $CeRhIn_5$, $CeIrIn_5$

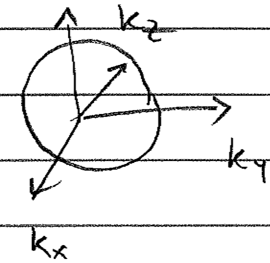
"115" materials

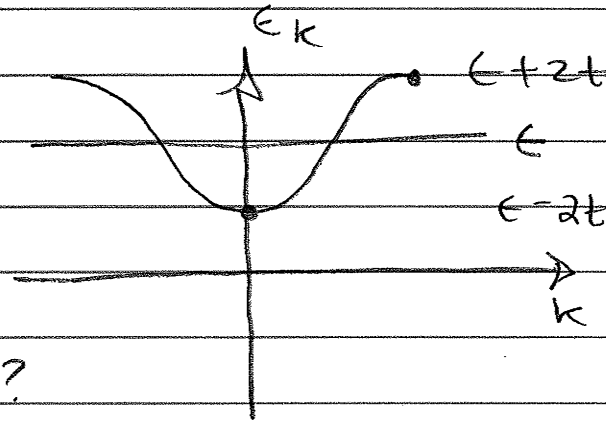
$N(E_F)$ is abnormally large → interesting physics

$$N(E) = \frac{V}{2\pi^2}\frac{m}{\hbar^2}\left(\frac{2mE}{\hbar^2}\right)^{1/2}$$

ascribe large $N(E)$ to large effective mass "heavy"

$N(E) \sim E^{1/2}$ ← more k points as Fermi sphere gets bigger





Do also our 1-d example

$\Delta = 0$ first

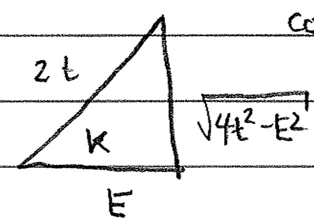
Where are there more k points?

Set $\epsilon = 0$ for simplicity

$N(E)\,dE = n(k)\,dk$

↑ flat one k point per $\frac{2\pi}{L}$

$E = -2t\cos k$



$\cos k = \frac{E}{2t}$

$dE = 2t\sin k\, dk$

$N(E)\,dE = \frac{L}{2\pi}\frac{dE}{2t\sin k}$

$\sin k = \frac{\sqrt{4t^2-E^2}}{2t}$

$= \frac{L}{2\pi}\frac{dE}{\sqrt{4t^2-E^2}}$

$N(E) = \frac{L}{2\pi}\frac{1}{\sqrt{4t^2-E^2}}$

peaks at $E = \pm 2t$

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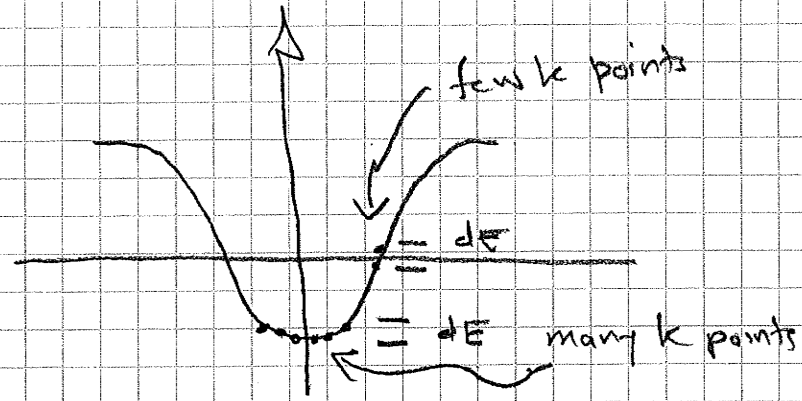
Physically:



what if $\Delta \neq 0$