MIDTERM EXAM

Physics 140B– SPRING 2012

Instructions: Do four of the six problems.

[**1.**] Consider a simple cubic lattice with lattice constant a. That is, the real space lattice vectors are $\vec{a}_1 = a\hat{x}, \quad \vec{a}_2 = a\hat{y}$, and $\vec{a}_3 = a\hat{z}$. What are the reciprocal lattice vectors $\vec{b}_i$? What is the first Brillouin zone? Draw the free electron energy bands $E(\vec{k}+\vec{G}) = \hbar^2|\vec{k}+\vec{G}|^2/2m$ folded back into the first Brillouin zone. For simplicity, set $(k_x, k_y, k_z) = (k, 0, 0)$ so that you are just plotting E as a function of a single variable k. Do at least the first seven smallest $\vec{G}$ (lowest energies).

[**2.**] Consider electrons hopping in one dimension with a dispersion relation $E(k) = -2t\cos k$. Compute the density of states. Connect any features you notice in $E(k)$ to the plot of $E(k)$ versus k.

[**3.**] Sketch the density of states for the dispersion relation $E(k_x, k_y) = -2t\,[\cos k_x + \cos k_y]$. What lattice gives this $E(k_x, k_y)$? What physical system (class of materials) do solid state physicists hope to describe with this $E(k_x, k_y)$? Why might it be a reasonable choice (that is, what characteristic of this class of materials makes one want to use this dispersion relation?) What does the Fermi surface look like for Fermi energy near the bottom of the band (E just a bit more than $-4t$)? What does the Fermi surface look like for $E = 0$?

[**4.**] Consider the second quantized Hamiltonian

$$\hat{H} = -t\sum_l (c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l) + \Delta \sum_l (-1)^l c_l^\dagger c_l$$

Write the matrix for $\hat{H}$ for one electron on an eight site lattice. That is, choose the occupation number basis $|\,1\,0\,0\,0\cdots 0\rangle$, $|\,0\,1\,0\,0\cdots 0\rangle$, $|\,0\,0\,1\,0\cdots 0\rangle$, $\cdots$ and act with $\hat{H}$ on each state, writing the result as a matrix. Use periodic boundary conditions. What are the eight eigenvalues for $\Delta = 0$? What are the eight eigenvalues for $t = 0$? What are the eight eigenvalues for general t, Δ? (This last question is not so easy.)

[**5.**] Solve for the eigenenergies and eigenfunctions of the Schroedinger equation for a one dimensional delta function potential $V(x) = +g\,\delta(x)$.

[**6.**] State and prove Bloch's Theorem.