project:
SP-1

Why specific heat?

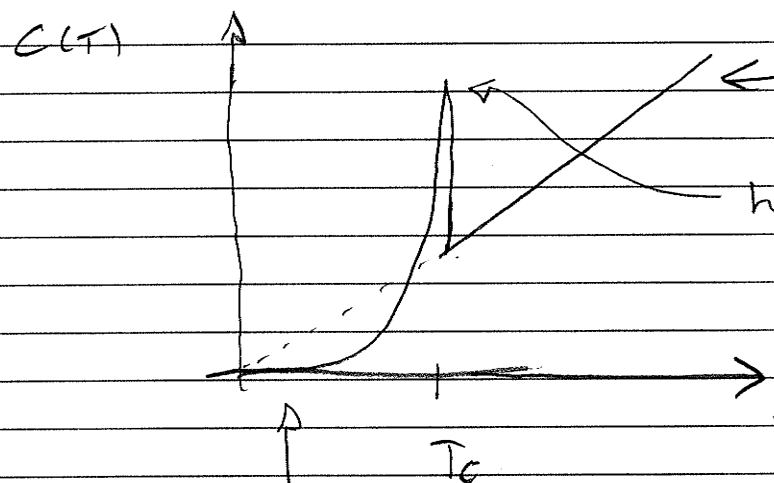
(1) A familiar example

$C/N = 5/2\, k_B$ diatomic ideal gas

$C/N = 3/2\, k_B$ Monatomic ideal gas

So you can tell gases apart.

(2) Less familiar: Superconductivity



← usual electronic γT

prediction of BCS
height of peak is 2.43 times
~~value just before peak~~
confirmed in expt

$C(T) \sim e^{-\Delta / k_B T}$

~~gives energy gap Δ, a crucial quantity~~

Δ = energy to break cooper pair,

bound state of $k\uparrow$ electron and $-k\downarrow$ electron

project:

SP-2

Even crucial to high T_c SC:

Δ_k

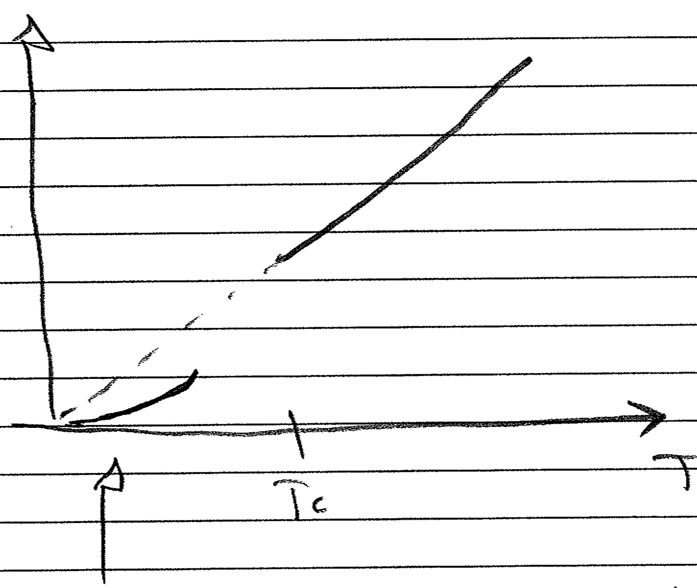
↑ energy to break $k\uparrow, -k\downarrow$ bound state

Conventional SC Δ_k is k independent Δ_0

High T_c Δ_k does depend on $\vec{k}$ and can vanish for certain momenta

$$\Delta_{\vec{k}} = \Delta_0 (\cos k_x - \cos k_y)$$ "d-wave"

Consequence



T^3 instead of $e^{-\Delta/k_B T}$

A KEY EXPT TO PROVE d-wave symmetry

DiVOGA Impt fact about high T_c.

project:

QSHO-1

Quantum Oscillator

Classical $E(x\,p) = \frac{1}{2} k x^2 + p^2/2m$

$$Z = \int dx \int dp \; e^{-\beta E}$$

$$= \int dx \int dp \; e^{-\frac{1}{2}k\beta x^2} \, e^{-\beta p^2/2m}$$

$$= \left(2\pi/k\beta\right)^{1/2} \left(2\pi m/\beta\right)^{1/2} \sim \beta^{-1}$$

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln Z = 1/\beta = k_B T$$

$$C = k_B$$

$$E_n = (n + \tfrac{1}{2})\hbar\omega$$

Amazingly Quantum Stat Mech same procedure as classical stat mech! Except extra initial step solve Sch Eqn $\to E_n$

$$Z = \sum_n e^{-\beta E_n} = e^{-\beta \frac{1}{2}\hbar\omega} \sum_{n=0}^{\infty} \left(e^{-\beta\hbar\omega}\right)^n$$

x^n

replaces $\int dx \int dp$

$$\sum_0^{\infty} x^n = 1/_{1-x}$$

project:

QSHO-2

$$Z = e^{-\beta \frac{1}{2}\hbar\omega} \left(1 - e^{-\beta\hbar\omega}\right)^{-1}$$

$$\langle E \rangle = -\frac{\partial}{\partial\beta} \ln Z = -\frac{\partial}{\partial\beta}\left[-\beta \tfrac{1}{2}\hbar\omega - \ln\left(1 - e^{-\beta\hbar\omega}\right)\right]$$

$$= \frac{1}{2}\hbar\omega + \frac{1}{1-e^{-\beta\hbar\omega}} e^{-\beta\hbar\omega}\hbar\omega$$

expected!

$$= \hbar\omega\left[\frac{1}{e^{\beta\hbar\omega}-1} + \frac{1}{2}\right]$$

$$\langle E \rangle = \hbar\omega\left[\langle n \rangle + \frac{1}{2}\right]$$

Bose-Einstein distribution

Consider high T (β small) ($k_B T \gg \hbar\omega$)

$$e^{\beta\hbar\omega} \simeq 1 + \beta\hbar\omega$$

$$\langle E \rangle \sim \hbar\omega\left[\frac{1}{\beta\hbar\omega} + \frac{1}{2}\right]$$

$$= \frac{1}{\beta} + \underbrace{\frac{1}{2}\hbar\omega}_{\phi}$$

$$= k_B T$$

Recover classical limit at high T

project:

QSHO - 3

We expect quantum $\xrightarrow[\text{systems}]{\text{large}}$ classical

quantum $\xrightarrow[\hbar \to 0]{}$ classical

Did we expect quantum $\to$ classical at high T

one argument $\hbar\beta$ enters so $T \to \infty$ $\beta \to 0$

equivalent to $\hbar \to 0$

Another way: Path integral $e^{-\beta(\hat{A}+\hat{B})}$

$$e^{-\beta(\hat{A}+\hat{B})} \neq e^{-\beta\hat{A}} e^{-\beta\hat{B}} \quad \text{if } [\hat{A},\hat{B}] \neq 0$$

But if $\beta \to 0$ it's a good approximation!

$$1 - \beta(\hat{A}+\hat{B}) + \frac{\beta^2}{2}(\hat{A}+\hat{B})^2$$

$$1 - \beta\hat{A} + \frac{\beta^2}{2}\hat{A}^2 \qquad 1 - \beta\hat{B} + \frac{\beta^2}{2}\hat{B}^2$$

Difference?

DiVOGA

project:

QSHO-31

Move on path integral

$$e^{-\beta\hat{H}} = e^{-L\epsilon\hat{H}}$$

$\beta = L\epsilon$

large, small

exact! since $[\hat{H},\hat{H}]=0$

$$= e^{-\epsilon\hat{H}} e^{-\epsilon\hat{H}} \cdots e^{-\epsilon\hat{H}}$$

$$\approx e^{-\epsilon\hat{A}} e^{-\epsilon\hat{B}} e^{-\epsilon\hat{A}} e^{-\epsilon\hat{B}} \cdots e^{-\epsilon\hat{A}} e^{-\epsilon\hat{B}}$$

if $\hat{H} = \hat{A} + \hat{B}$

good approximation if $\epsilon \to 0$ (large L)

(like Riemann sum $\to$ to integral)

project:

QSHO-4

So far considered a *single* harmonic oscillator with frequency ω. But we know we have whole family of oscillators $\omega(q) = \frac{2k}{m}[1-\cos q]$ in 1 d.

Debye model $\omega(q) = v q$ (phonons) $\forall q$

Q: Why might that be reasonable at low T?

A: Only small ω occupied.

$$E \sim \int_0^\infty \underbrace{q^2}_{3D} dq \, \frac{\omega}{e^{\beta\hbar\omega}-1} \sim \int_0^\infty \frac{q^3\, dq}{e^{\beta\hbar v q}-1}$$

Change variables $x = \beta\hbar v q$ $\quad q = \frac{x T k_B}{\hbar v}$

$$E \sim T^4 \underbrace{\int_0^\infty \frac{x^3\, dx}{e^x - 1}}_{\text{some } \#}$$

$E \sim T^4$

$\boxed{C \sim T^3}$ Key CM physics result

phonons contribute T^3 to specific heat of solid.

project:

ALT-1

Alternate definition of entropy / "derivation" of Boltzmann

$$S = -k_B \sum_s p_s \ln p_s$$

Maximize Free energy

$$F = \langle E \rangle - TS$$

$$F = \sum p_s E_s + k_B T \sum p_s \ln p_s$$

Maximize F

$$\partial F / \partial p_s = E_s + k_B T \ln p_s + T = 0$$

$$p_s \propto e^{-E/k_B T} \leftarrow \text{Boltzmann}$$