project:

Fermi's Golden Rule

$$\Gamma_{i \to f} = \frac{2\pi}{\hbar} \left| \langle f | \hat{V} | i \rangle \right|^2 \delta(E_f - E_i)$$

Rate of transition from state $|i\rangle$ to state $|f\rangle$

caused by perturbation $\hat{V}$

Unfamiliar Eqn

(1) Check dimensions. units of $\langle f | \hat{V} | i \rangle \to$ Energy

$\delta(E_f - E_i) \to 1/\text{Energy}$

so units of $\Gamma \sim \text{Energy}/\hbar \sim \text{frequency} \sim 1/\text{time}$ ✓

(2) Anything similar?

2nd order perturbation theory $\quad E_n^{(2)} = \sum_m \frac{\left| \langle m | \hat{V} | n \rangle \right|^2}{E_n^0 - E_m^0}$

Indeed Fermi's golden Rule derived from

<u>time</u> <u>dependent</u> perturbation theory

(3) Physically plausible: For a potential $\hat{V}$ to cause a transition from $|i\rangle$ to $|f\rangle$ it ought to "mix" or "connect" the states ie $\langle f | \hat{V} | i \rangle \neq 0$

project:
FGR-1

Fermi Golden Rule

Explanation of Bragg Scattering

Recall perturbation theory

Question how does E_n^0 defined by

$$H^0 \psi_n^0 = E_n^0 \psi_n^0$$

change when $H^0 \to H^0 + V$ ← perturbation

Obviously must depend on V and ψ_n^0. Simplest guess is

$$E_n^1 = \langle \psi_n^0 | V | \psi_n^0 \rangle = \int dr\, \psi_n^0(r) V(r) \psi_n^0(r)$$

which is correct. What would we guess for second order correction to energy? Could be, for example

$$E_n^2 \overset{?}{=} \langle \psi_n^0 | V^2 | \psi_n^0 \rangle = \int dr\, \psi_n^0(r)\, V^2(r)\, \psi_n^0(r)$$

But this is not dimensionally correct! There must be an energy in denominator e.g.

$$E_n^2 = \langle \psi_n^0 | V^2 | \psi_n^0 \rangle / E_n^0$$

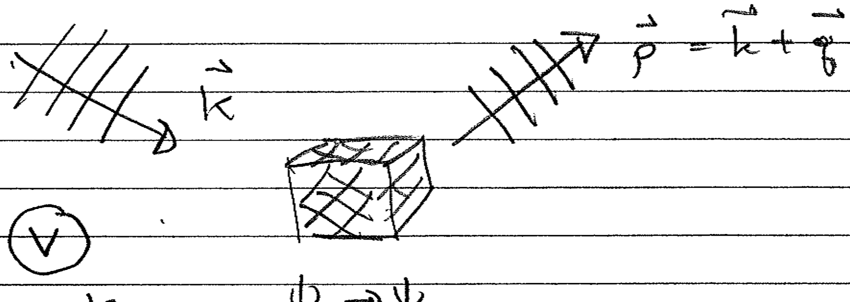
But correct answer is (see Physics 115B)

$$E_n^2 = \sum_{m \neq n} |\langle \psi_n^0 | V | \psi_m^0 \rangle|^2 / E_n^0 - E_m^0$$

project:

FGR-2

Let's think about a possible formula for the scattering of light (or some sort of incident wave) by an object



describes interaction of incident wave with object

$\psi_i \to \psi_f$

State of object changes from ψ_i to ψ_f

Rate of scattering. How do we get rate (units of $1/T$)

$1/T \to \text{Energy}/\hbar \leftarrow$ planck's constant.

So a guess would be $\frac{1}{\hbar} \langle e^{i\vec{k}\cdot\vec{r}} \psi_i | V | e^{i\vec{p}\cdot\vec{r}} \psi_f \rangle$

This is not quite right. One thing missing is idea of

energy conservation $\delta(E_k + E_i - E_p - E_f)$

delta function units of $1/\text{Energy}$ ($1/\text{argument}$)

$$\boxed{\text{Fermi's Golden Rule} \qquad \Gamma = \frac{2\pi}{\hbar} \left| \langle e^{i\vec{k}\cdot\vec{r}} \psi_i | V | e^{i\vec{p}\cdot\vec{r}} \psi_f \rangle \right|^2}$$

perhaps the most famous and useful formula of all QM!

project: FGR-3

Suppose the state of the object doesn't change $\psi_i = \psi_f$

get $$\Gamma = \frac{2\pi}{\hbar} \langle e^{ik\cdot r} | V | e^{ip\cdot r} \rangle \, \delta(E_k - E_p)$$

Energy of incident and outgoing radiation is same "elastic scattering"

$$\int d^3r \, e^{-ik\cdot r} V(r) e^{i\vec{p}\cdot\vec{r}}$$

$$= \int d^3r \, e^{i\vec{q}\cdot\vec{r}} V(r) = V(q)$$

$$\Gamma = \frac{2\pi}{\hbar} |V(\vec{q})|^2 \, \delta(E_k - E_p)$$

sort of reasonable
scattering rate $k \to p$
depends on Fourier component
of V of wave vector $q = p - k$

This is in fact exactly what we will be using in our analysis of x-ray scattering

The analysis/argument of these 3 pages are a more rigorous ~~a~~ background.