DRUDE MODEL

So far pictured electrons in metal as collection of noninteracting quantum particles in a box. Almost like ideal gas, except Pauli makes big changes

Maxwell-Boltzmann → Fermi Dirac

<KE> much higher

Pauli Blocking only $kT/E_F \sim 1/100$ of e^- participate in $C(T)$

Example of more general principle in CM only electrons near Fermi surface matter Vast simplification of theory made possible

Drude Model retains picture of noninteracting "gas" of electrons, but adds collisions with impurities.

If there were no impurities in a solid and no phonons resistance would be zero.

(1) Between collisions e^- move as free particles

(2) collisions instantaneous and occur with probability dt/τ indep. of e^- location or velocity

DC conductivity in Drude Model

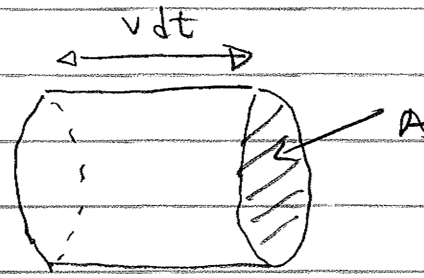
$E = \rho j$ $\qquad\qquad$ $j = \sigma E$

Electric field $\qquad$ resistivity $\qquad$ current density I/A $\qquad$ conductivity

You have seen this before : $V = IR$

$EL = \rho L \frac{I}{A}$ $\qquad\qquad$ $R = \rho L/A$ $\qquad$ (9c)

$j = -nev$



charge flow through A

is $-enA\,v\,dt = dq$ $\qquad\qquad$ n electrons /volume

$I = dq/dt = -enAv$ $\qquad\qquad$ $j = -nev$

e^- at time ϕ : velocity $\vec{V}_0$ ← random

just after a collision

time t later : velocity $\vec{v}_0 - \frac{e\vec{E}t}{m}$

Average velocity is $\frac{-e\vec{E}\langle t\rangle}{m}$ since $\langle\vec{v}_0\rangle = 0$

$\langle t\rangle = \tau$ if prob of collision in dt is dt/τ

$$\langle\vec{v}\rangle = \frac{-e\vec{E}\tau}{m}$$

$$\vec{j} = \left(\frac{ne^2\tau}{m}\right)\vec{E}$$

$\boxed{\sigma = ne^2\tau/m \text{ in DRUDE model}}$

Since σ can be measured and n, e^2, m known

can get τ $\tau \sim 10^{-14} - 10^{-15}$ sec for typical metals

reasonable?!

However, can purify metals and vastly increase σ

(increase τ).

$\vec{E}$ and $\vec{B}$ and t dependent!

Now do somewhat more general analysis

$\vec{p}(t)$ = momentum at time t

$$\vec{p}(t+dt) = \underbrace{\left(1-\frac{dt}{\tau}\right)}_{\text{fraction of uncollided electrons}}\Big[\vec{p}(t)+\underbrace{\vec{f}(t)}_{\text{force}}dt\Big] + o(dt^2)$$

(collided electrons are fraction dt/τ and any effect of $\vec{f}$ on them is $o(f\,dt)$ so total is dt^2)

$$\vec{p}(t+dt) = \vec{p}(t) + \vec{f}(t)\,dt - \frac{dt\,\vec{p}(t)}{\tau}$$

$$d\vec{p}/dt = \frac{-\vec{p}(t)}{\tau} + \vec{f}(t)$$

remind you of anything?

Mass on spring with friction $f_{fric} = -\gamma v = -\gamma p/m$

$$dp/dt = -\gamma\, p/m \underbrace{- kx}_{F_{spring}}$$

So $\frac{1}{\tau}$ like $\frac{\gamma}{m}$ (friction)

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Or a RLC circuit

$$\frac{dI}{dt} = -\frac{IR}{L} - \frac{Q}{LC}$$

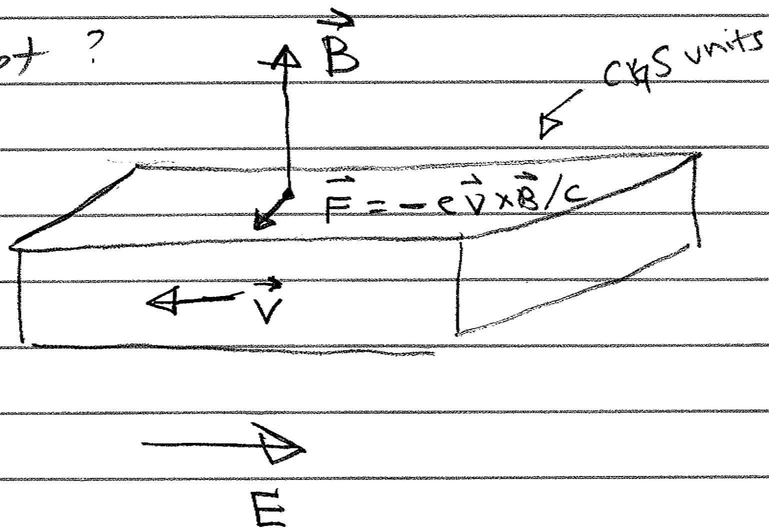
again $\frac{1}{\tau}$ like $\frac{R}{L}$

Apply $d\vec{p}/dt = -\vec{p}/\tau + \vec{f}$

to HALL EFFECT (combined $\vec{E} + \vec{B}$)

Interesting science "story". What did Hall expect to see

going in to expt ?



Hall Expected:

Electrons pile up on one side of wire → resistance increases (e$^-$ more crowded together).

He did not see this!

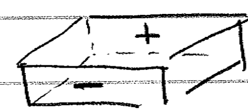
Why not?

Electrons do start piling up on one side but resulting charge gradient produces $\vec{E}$ field which cancels $\vec{B}$.

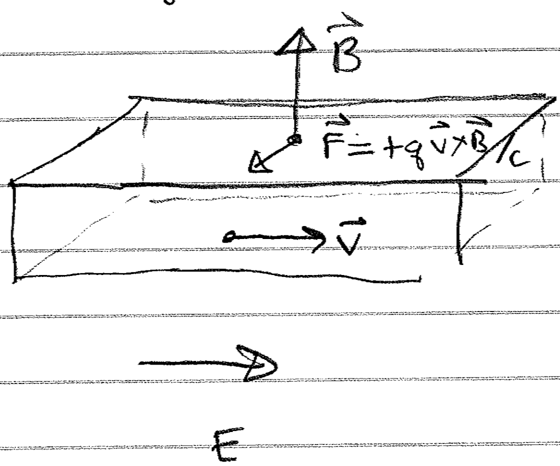
Hall voltage

Also very interesting is that Hall voltage determines sign of charge carriers in a solid. This was not known for certain at the time (1879)

on page 5 (negative charge carriers) pile up on close face so



If charge carriers (+) $\vec{F}$ is same direction and + carriers pile up on close side



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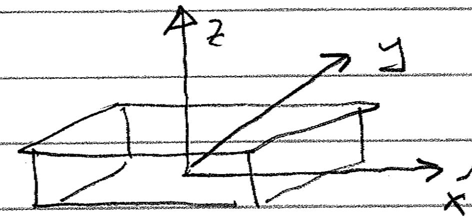
$\underbrace{\vec{v}\times\vec{B}}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ p_x & p_y & p_z \\ 0 & 0 & B \end{vmatrix}$$

$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{p}}{mc}\times\vec{B}\right) - \vec{p}/\tau$$



steady state LHS = 0

$$0 = -e\left(E_x + \frac{B p_y}{mc}\right) - p_x/\tau$$

$$0 = -e\left(E_y - \frac{B p_x}{mc}\right) - p_y/\tau$$

multiply by $\frac{ne\tau}{m}$

$$\left(\frac{ne^2\tau}{m}\right)E_x = \frac{eB}{mc}\tau j_y + j_x$$

Recall

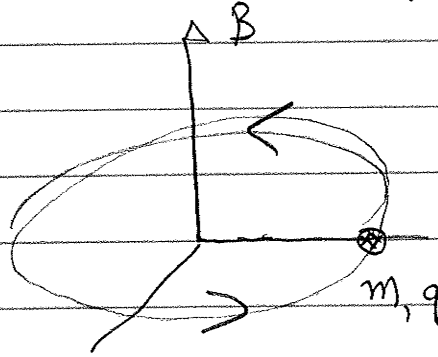
$j = -nev$

$= -ne\,p/m$

σ

$$\left(\frac{ne^2\tau}{m}\right)E_y = -\frac{eB}{mc}\tau j_x + j_y$$

recognize??



mass m, charge q

circular path in $\vec{B}$ field

since $\vec{F} = q\frac{\vec{v}\times\vec{B}}{c}$ ⊥ to $\vec{v}$

$$q\frac{vB}{c} = m\frac{v^2}{R}$$

$$\frac{qB}{mc} = \frac{v}{R} = \frac{2\pi}{T} = \omega$$

cyclotron frequency

$$\sigma E_x = \omega\tau J_y + J_x$$

$$\sigma E_y = -\omega\tau J_x + J_y$$

If $J_y = 0$ (equil)

$\sigma E_x = J_x$

$\sigma E_y = -\omega\tau J_x$

"Hall Coefficient" $R_H \equiv \dfrac{E_y}{j_x B} = \dfrac{-\omega\tau}{\sigma B} = \dfrac{-\frac{eB}{mc}\frac{\sigma m}{ne^2}}{\sigma B}$

$$R_H = -\frac{1}{nec} \qquad ne_cR_H = -1$$

!! Depends only on carrier density

alkali Metals	$-1/ne_cR_H$
Li	0.8
Na	1.2
K	1.1
Rb	1.0
Cs	0.9
Cu	1.5
Ag	1.3
Be	−0.2
Mg	−0.4
Al	−0.3

!!

AC conductivity!

Let's introduce some time dependence (but just $\vec{E}$ field)

$$\vec{E}(t) = \mathrm{Re}\left(\vec{E}\, e^{i\omega t}\right)$$

as usual guess

$$\vec{p}(t) = \mathrm{Re}\left(\vec{p}\, e^{i\omega t}\right)$$

$d\vec{p}/dt = -\vec{p}/\tau + \vec{f}$

$$-i\omega \vec{p} = -\vec{p}/\tau - e\vec{E} \qquad (1/\tau - i\omega)\vec{p} = -e\vec{E}$$

$$\vec{j} = \frac{-ne\vec{p}}{m} = \frac{ne^2/m\ \vec{E}}{1/\tau - i\omega}$$

$\omega = 0$ recover $\sigma = \frac{ne^2\tau}{m}$

$$\sigma(\omega) = \frac{ne^2\tau/m}{1 - i\omega\tau}$$