PROBLEM SET 4 Due Friday October 21

Physics 115B– FALL 2011

Analytic:

[**1.**] Griffiths Problem 4.13

[**2.**] Griffiths Problem 4.14

[**3.**] Griffiths Problem 4.16

[**4.**] The diffusion equation decribes the spreading of particles, heat, etc and is sometimes referred to as the 'imaginary time Schroedinger equation' since the two look very similar with the replacement $t \leftrightarrow it$. In the absence of a source, the diffusion equation is,

$$D\,\nabla^2\Psi(\vec{r},t) = \frac{\partial}{\partial t}\Psi(\vec{r},t)$$

D is the 'diffusion constant'. When D is large, the spreading is rapid. Solve the diffusion equation in one dimension for $\Psi(x,t)$ given $\Psi(x,t=0)=\delta(x)$. Also, give the general solution for any $\Psi(x,t=0)$. Hint: The mathematics is basically identical to the discussion of the free particle Schroedinger equation in Griffiths Sec. 2.4.

[**5.**] Now consider the diffusion equation in 3D, with a source,

$$(D\nabla^2+\lambda)\Psi(\vec{r},t) = \frac{\partial}{\partial t}\Psi(\vec{r},t)$$

The λ term represents, for example, a uniform source of heat in a material, and $\Psi(\vec{r},t)$ would be the temperature at position $\vec{r}$ and time t. In this problem, we are going to see under what conditions a block of material will melt. The physical picture is that the $D\nabla^2$ term helps prevent melting by diffusing the heat away, whilst the λ term tries to increase the temperature. We need to see which effect wins. Begin to solve this diffusion equation by separating out the time dependence $\Psi(\vec{r},t)=R(\vec{r})g(t)$. How does $g(t)$ differ from the $e^{-iEt/\hbar}$ encountered for the Schroedinger equation? Finish solving the equation for a cubical block of material $0<x,y,z<a$. What is the melting condition? Hint: You need to keep $g(t)$ from diverging!

[**6.**] Solve the 3D diffusion equation with a source, as in problem 5, but for a sphere of material of radius $r=a$ instead of a cube. This problem is basically identical to the spherical well Schroedinger equation discussed in class. Again, determine the melting condition. Compare with the melting condition of Problem 5. Which is more prone to melt, a sphere of volume V or a cube of volume V? Why? Comment: Problems 5,6 have an important application, namely to the computation of the critical shape and mass of uranium needed for the core of a power plant to melt.

Numeric:

Comment: For the first part of the course, as you develop skill in programming, the computational problems will not necessarily have anything to do with quantum mechanics.

[7.] Write a C or C++ program to solve the Kepler problem by molecular dynamics. You need to keep track of position and velocity in two dimensions. The heart of your code is

$$
\begin{aligned}
x &= x + v_x \, dt \\
y &= y + v_y \, dt \\
r &= \sqrt{x^2 + y^2} \\
v_x &= v_x - \frac{G M_{\rm sun} \, x}{r^3} \, dt \\
v_y &= v_y - \frac{G M_{\rm sun} \, y}{r^3} \, dt
\end{aligned}
$$

Explain where the equations for v_x and v_y come from. Why is there an r^3 in the denominator?! Generate a few orbits (e.g. circular, parabolic) to test your code.