Spherical Coordinates

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

VECTOR:
$|\vec{A}|$ and direction
— or —
Components A_x, A_y, A_z

Actually several options here depending on choice of basis vectors "usual" one is $\hat{x}, \hat{y}, \hat{z}$

In QM also $|\psi\rangle$ → vector like $\vec{A}$

$\langle x|\psi\rangle = \psi(x)$
$\langle p|\psi\rangle = \tilde{\psi}(p)$ } different bases

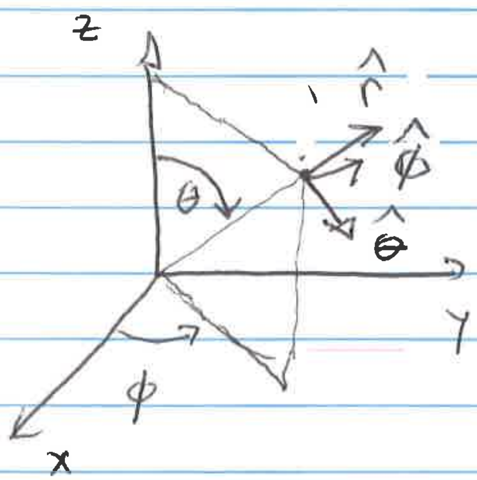
$|\psi(x)|^2$ = prob @ position x

$|\tilde{\psi}(p)|^2$ = prob of momentum p

But for many problems other basis vector choices are more convenient

$$\vec{V} = C\frac{\hat{r}}{r^2} = \frac{C\vec{r}}{r^3}$$ ← we discussed this before and in your HW

$$= C\frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^{3/2}}$$ ← MUCH UGLIER!



$z = r\cos\theta$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

What are other basis vectors?

$\hat{\theta}, \hat{\phi}$

$\hat{r}, \hat{\theta}, \hat{\phi}$ are $\perp$

See text for expressions for $\hat{\theta}, \hat{\phi}$ in terms of $\hat{x}\,\hat{y}\,\hat{z}$

IMPT: $\hat{r}, \hat{\theta}, \hat{\phi}$ depend on location in space!

↖ a price you pay

$$d\vec{l} = \hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$$

$$\rightarrow \hat{r}\,dr + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

must be so dimensionally

again a price in greater complexity

$$dx\,dy\,dz \Leftarrow d\tau \Rightarrow r^2 \sin\theta\,dr\,d\theta\,d\phi$$

What about derivatives?

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\sin\theta\frac{\partial}{\partial \theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \phi^2}$$

MESSY, BUT BETTER THAN TRYING TO USE x, y, z

WILL SEE IN QM $\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{r}\right]\psi(r,\theta,\phi) = E\,\psi(r,\theta,\phi)$

↑ scalar field

H atom Schroedinger Eqn ↗

look up cylindrical coordinates in text!