$\boxed{\text{Gauge Freedom and Deep Physics}}$

changes which do
not affect eqns
of motion and
hence path of particle

$V(y) = mgy$

$F_y = -\dfrac{dV}{dy} = -mg$

$y \to y + y_0$

leaves F_y unchanged

Similarly

$\vec{E} = -\vec{\nabla}\phi \qquad \phi \to \phi + c$

$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{A} \to \vec{A} + \vec{\nabla}\Lambda$

QM $\qquad \psi(x) \to e^{i\alpha}\psi(x)$

$|\psi(x)|^2$ unchanged

But what if $\quad e^{i\alpha(x)}\psi(x)$ still same $|\psi(x)|^2$

But $\quad -\dfrac{\hbar^2}{2m}\dfrac{d^2}{dx^2}\psi(x)$

act on $\alpha(x)$

∇^2 in 3D

$$i\hbar\vec{\nabla} \rightarrow i\hbar\vec{\nabla} - \frac{e}{c}\vec{A}$$

$$\left.\begin{aligned} \vec{A} &\Rightarrow \vec{A}' = \vec{A} + \frac{c\hbar}{e}\nabla\alpha \\ \psi(\vec{r}) &\Rightarrow e^{i\alpha(\vec{r})}\psi(r) \end{aligned}\right\}$$ leave Schroedinger Eqn invariant

Demanding QM be invariant under local changes of phase

$\Longrightarrow$ Must introduce EM fields!!!

Similarly with more complex high energy theory

KD-1

Kronecker Delta

$$\delta_{nm} = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases} \qquad n, m \text{ integers}$$

(1) Set of orthonormal vectors $\{\vec{v}_n\}$

$$\vec{v}_n \cdot \vec{v}_m = \delta_{nm}$$

orthogonal $\rightarrow 0 \quad n \neq m$

normalized $\rightarrow 1 \quad n = m$

(2) Elements of identity matrix $\mathbb{I}_{nm} = \delta_{nm}$

(3) Components of basis vectors

$$\hat{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \hat{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$(\hat{e}_n)_m = \delta_{nm}$$

Many many cases where δ_{nm} arises