

PHYSICS 110A, WINTER 2017
ELECTRICITY AND MAGNETISM

Assignment Six, Due Friday, March 2, 5:00 pm.

[1.] A hydrogen atom acts as if it had an electrostatic potential

$$\phi(r) = \frac{q}{4\pi\epsilon_0 r} \left(1 + \frac{r}{a_0}\right) e^{-2r/a_0},$$

where q is the charge on the proton and $a_0 = \hbar^2/m_e q^2 = 0.529\text{\AA}$ is the Bohr radius.

- (a) Find the corresponding charge density and interpret the various terms physically.
- (b) Verify by *explicit* integration that your resulting charge density from part (a) indeed produces the original potential.
- (c) What is the net charge inside a sphere of radius a_0 ? What is the electric field strength at this distance?

[2.] Solve for the potential in the region between two concentric spherical shells of radii a and b , given the potentials $V_a(\theta)$ and $V_b(\theta)$. Your objective should be to write the coefficients of an expansion of the potential as integrals involving the (unknown) functions V_a and V_b . Choose some specific form of the functions that you find particularly amusing (and not too hard!) and do the integrals.

[3.] A sphere of radius R has a potential $V(r, \theta) = V_0 \cos^2 \theta$ on its surface. Determine the potential outside the sphere.

[4.] Griffiths 3-15.

[5.] Griffiths 3-21.

[6.] Griffiths 3-23.

[7.] Griffiths 3-14.

NB: Solns are in CGS units.

3. Hydrogen atom has potential $\Phi(r) = \frac{q}{r} (1 + r/a_0) e^{-2r/a_0}$ where q is charge on proton and $a_0 = \hbar^2/m_e e^2 \approx 0.529 \text{ \AA}$ is Bohr radius.

a) Find $\rho(r)$ and interpret physically.

We know that $\nabla^2 \Phi(r) = -4\pi\rho(r)$. In a case like this, there is no θ and ϕ dependence and the Laplacian is $\frac{1}{r} \frac{d^2}{dr^2} r$

$$\begin{aligned} \text{Thus for } r \neq 0 \quad \rho(r) &= -\frac{1}{4\pi} \frac{1}{r} \frac{d^2}{dr^2} q (1 + r/a_0) e^{-2r/a_0} \\ &= -\frac{q}{4\pi r} \frac{d}{dr} \left(\frac{1}{a_0} e^{-2r/a_0} + \frac{-2}{a_0} (1 + r/a_0) e^{-2r/a_0} \right) \\ &= -\frac{q}{4\pi r} \left(-\frac{2}{a_0} e^{-2r/a_0} + \frac{4}{a_0} \left(1 + r/a_0 \right) e^{-2r/a_0} - \frac{2}{a_0} e^{-2r/a_0} \right) \\ \rho(r) &= -\frac{q}{\pi a_0^3} e^{-2r/a_0} \quad \text{for } r \neq 0 \end{aligned}$$

when $r=0$ we must recognize $\nabla^2 \frac{1}{r} = -4\pi \delta(\vec{r})$ so that we get an extra term

$$\rho_p(r) = -\frac{1}{4\pi} -4\pi q \delta(\vec{r}) = q \delta(\vec{r})$$

All together $\rho(\vec{r}) = q \delta(\vec{r}) - \frac{q}{\pi a_0^3} e^{-2r/a_0}$

The physical interpretation is clear. The first term is the charge density of the proton which is localized exactly at $r=0$. The second term is the charge density of the electron "cloud" around the proton. In fact, the ground state wave function is $\psi_0(r) = \frac{1}{\sqrt{\pi} a_0} e^{-r/a_0}$ (Baym) so that $|\psi_0(r)|^2 = \frac{1}{4\pi} \frac{4}{a_0^3} e^{-2r/a_0} = \frac{1}{\pi a_0^3} e^{-2r/a_0}$. Thus the second term is simply $-q |\psi_0(r)|^2$ as expected.

2.
1-2

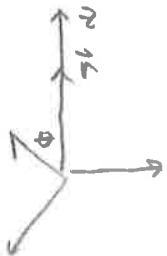
We might also comment that we expect the atom to be electrically neutral overall. So we verify that

$$\begin{aligned}
 Q_{\text{TOT}} &= \int \rho(\vec{r}) dV = q + \int_0^{\infty} 4\pi r^2 \frac{-b}{\pi a_0^3} e^{-2r/a_0} dr \\
 &= q \left(1 - \frac{4}{a_0^3} \int_0^{\infty} r^2 e^{-2r/a_0} dr \right) \quad \text{let } u = 2r/a_0 \quad dr = a_0/2 du \\
 &= q \left(1 - \frac{4}{a_0^3} \left(\frac{a_0}{2}\right)^3 \int_0^{\infty} u^2 e^{-u} du \right) \quad \text{But } \int_0^{\infty} u^n e^{-u} du = n! \\
 &= q \left(1 - \frac{1}{2} 2! \right) = 0
 \end{aligned}$$

Thus $Q_{\text{TOT}} = 0$
 The total charge density integrates to zero - the H atom is electrically neutral.

(b) verify by explicit integration that $\rho(r)$ reproduces $\Phi(r)$.

$$\Phi(\vec{r}) = \int \frac{\rho(\vec{r}') d\vec{r}'}{|\vec{r} - \vec{r}'|}$$



choose for convenience \vec{r} to lie along \hat{z} -axis. Then
 $|\vec{r} - \vec{r}'|^2 = r^2 - 2rr' \cos\theta + r'^2$ and hence

$$\begin{aligned}
 \Phi(r) &= q \int \frac{\rho(\vec{r}') = \frac{1}{\pi a_0^3} e^{-2r'/a_0}}{\sqrt{r^2 - 2rr' \cos\theta + r'^2}} d\vec{r}' \quad \text{where } \delta(\vec{r}') = \delta(r') \delta(\theta') \delta(\phi') \frac{1}{r'^2 \sin\theta'} \\
 & \quad d\vec{r}' = r'^2 \sin\theta' d\theta' d\phi' dr' \\
 &= q \int_0^{\infty} r'^2 dr' \int_0^{2\pi} d\phi' \int_0^{\pi} \sin\theta' d\theta' \left\{ \frac{\delta(r') \delta(\theta') \delta(\phi')}{r'^2 \sin\theta'} - \frac{1}{\pi a_0^3} e^{-2r'/a_0} \right\} \frac{1}{\sqrt{r^2 - 2rr' \cos\theta + r'^2}} \\
 &= q/r - \frac{2\pi q}{\pi a_0^3} \int_0^{\infty} r'^2 dr' \frac{1}{rr'} \sqrt{r^2 - 2rr' \cos\theta + r'^2} \int_0^{\pi} e^{-2r'/a_0} \\
 \Phi(r) &= q/r \left\{ 1 - \frac{2}{a_0^3} \int_0^{\infty} dr' r' \left(\frac{r+r'}{r} - \frac{r-r'}{r} \right) e^{-2r'/a_0} \right\} \\
 &= q/r \left\{ 1 - \frac{2}{a_0^3} \left[\int_0^r dr' r' 2r' e^{-2r'/a_0} + \int_r^{\infty} dr' r' 2r e^{-2r'/a_0} \right] \right\}
 \end{aligned}$$

3.
1-3

$$\begin{aligned} \text{Now } \int_0^r r' e^{-2r'/a_0} dr' &= -r' a_0/2 e^{-2r'/a_0} \Big|_0^r + a_0/2 \int_0^r e^{-2r'/a_0} dr' \\ &= -r a_0/2 e^{-2r/a_0} + a_0/2 \left(\frac{a_0}{2} \right) e^{-2r'/a_0} \Big|_0^r \\ &= -r a_0/2 e^{-2r/a_0} + a_0^2/4 - a_0^2/4 e^{-2r/a_0} \end{aligned}$$

$$\begin{aligned} \text{and so } \int_0^r r'^2 e^{-2r'/a_0} dr' &= +r'^2 \frac{a_0}{2} e^{-2r'/a_0} \Big|_0^r + \frac{a_0}{2} \int_0^r r' e^{-2r'/a_0} dr' \\ &= -a_0 r^2/2 e^{-2r/a_0} + 4 a_0 \left(-r a_0/2 e^{-2r/a_0} + a_0^2/4 - a_0^2/4 e^{-2r/a_0} \right) \end{aligned}$$

$$\begin{aligned} \text{Also } \int_r^\infty dr' r' e^{-2r'/a_0} &= -\frac{a_0}{2} r' e^{-2r'/a_0} \Big|_r^\infty + a_0/2 \int_r^\infty e^{-2r'/a_0} dr' \\ &= \frac{a_0}{2} r e^{-2r/a_0} - a_0^2/4 e^{-2r/a_0} \Big|_r^\infty = \frac{a_0}{2} r e^{-2r/a_0} + \frac{a_0^2}{4} e^{-2r/a_0} \end{aligned}$$

Thus, all together,

$$\begin{aligned} \phi(r) &= q/r \left\{ 1 - \frac{2}{a_0^3} \left[+a_0 r^2 - a_0^2 r - a_0^3/2 + a_0 r^2 + a_0^2/2 r \right] e^{-2r/a_0} \right. \\ &\quad \left. - \frac{2}{a_0^3} \left[\frac{a_0^3}{2} \right] \right\} \\ &= q/r \left(-\frac{2}{a_0^3} \right) \left(-\frac{a_0^2}{2} r - \frac{a_0^3}{2} \right) e^{-2r/a_0} \end{aligned}$$

$$\phi(r) = q/r (1 + r/a_0) e^{-2r/a_0}$$

as required.

(e) calculate net charge in sphere of radius a_0

$$\begin{aligned} Q(a_0) &= \int_0^{a_0} 4\pi r^2 dr \left(\frac{q(r)}{4\pi r^2} - \frac{1}{4a_0^3} e^{-2r/a_0} \right) \\ &= q - \frac{4q}{a_0^3} \int_0^{a_0} r^2 e^{-2r/a_0} dr = q - \frac{4q}{a_0^3} \left\{ r^2 \frac{a_0}{2} e^{-2r/a_0} \Big|_0^{a_0} + a_0 \int_0^{a_0} r e^{-2r/a_0} dr \right\} \\ &= q \left(1 - \frac{4}{2} \left(-\frac{a_0^3}{2} e^{-2} + a_0^2 r e^{-2r/a_0} \Big|_0^{a_0} + \frac{a_0^2}{2} \int_0^{a_0} e^{-2r/a_0} dr \right) \right) \end{aligned}$$

1-4.

Scale then

$$\begin{aligned}
 Q(a_0) &= q \left(1 - \frac{4}{a_0^3} \left(-\frac{a_0^3}{2e^2} - \frac{a_0^3}{2e^2} + \frac{a_0^3}{4} e^{-2r/a_0} \Big|_0^{a_0} \right) \right) \\
 &= q \left(1 - \frac{4}{a_0^3} \left(-\frac{a_0^3}{e^2} + \frac{a_0^3}{4e^2} + \frac{a_0^3}{4} \right) \right) \\
 &= q \left(1 + \frac{4}{e^2} + \frac{1}{e^2} + 1 \right) = \frac{5q}{e^2}
 \end{aligned}$$

$$Q(a_0) = \frac{5}{e^2} q = .677 q$$

One way to look at this is to say a sphere of radius a_0 contains no proton charge $+q$ and $(1 - 5/e^2) = 32.3\%$ of the total electron charge $-q$.

The electric field strength at a_0 is given by Gauss' law

$$E(a_0) \frac{4\pi a_0^2}{e^2} = \frac{4\pi Q(a_0)}{e^2}$$

$$E(a_0) = \frac{5}{e^2} \frac{q}{a_0^2}$$

In MKS units $E(a_0) = \frac{5}{e^2} \frac{1.609 \cdot 10^{-19}}{(5.29 \cdot 10^{-10})^2} 9 \cdot 10^9 = 3.5 \cdot 10^{11} \frac{\text{volts}}{\text{m}}$

$$E(a_0) = 3.5 \cdot 10^9 \frac{\text{volts}}{\text{cm}}$$

According to Jackson p820 $1 \text{ volt/m} = \frac{1}{3} \cdot 10^{-4} \frac{\text{statvolt}}{\text{cm}}$

So that $E(a_0) = 1.17 \cdot 10^7 \frac{\text{statvolt}}{\text{cm}}$

This can also be computed directly:

$$E(a_0) = \frac{5}{e^2} \frac{q}{a_0^2} = \frac{5}{e^2} \frac{4.8 \cdot 10^{-10} \text{ statcoul.}}{(5.29 \cdot 10^{-9} \text{ cm})^2} = 1.17 \cdot 10^7 \frac{\text{statvolt}}{\text{cm}}$$

[2] The general soln of Laplace's eqn in spherical coordinates obeying azimuthal symmetry, as in problem 1, is

$$V(r, \theta) = \sum_{l=0}^{\infty} [A_l r^l + B_l r^{-(l+1)}] P_l(\cos \theta)$$

In contrast to problem 1, since we are confined to $a < r < b$ we cannot let $r \rightarrow \infty$ (or $r \rightarrow 0$) so we cannot conclude $A_l = 0$ (or $B_l = 0$).

Fortunately, we have two functions $V_a(\theta)$ and $V_b(\theta)$ whose information is sufficient.

$$V_a(\theta) \equiv V(a, \theta) = \sum_{l=0}^{\infty} [A_l a^l + B_l a^{-(l+1)}] P_l(\cos \theta)$$

$$V_b(\theta) \equiv V(b, \theta) = \sum_{l=0}^{\infty} [A_l b^l + B_l b^{-(l+1)}] P_l(\cos \theta)$$

From the orthogonality of the Legendre polynomials

$$\int_{-1}^1 P_n(x) P_m(x) dx = \frac{2\delta_{nm}}{(2n+1)}$$

or, in terms of $x = \cos \theta$

$$\int_0^{\pi} P_n(\cos \theta) P_m(\cos \theta) \sin \theta d\theta = \frac{2\delta_{nm}}{(2n+1)}$$

If we multiply * by $P_n(\cos \theta) \sin \theta$ and integrate,

$$\int_0^\pi V_a(\theta) P_n(\cos\theta) \sin\theta d\theta = (A_n a^n + B_n a^{-(n+1)}) \frac{2}{2n+1}$$

$$\int_0^\pi V_b(\theta) P_n(\cos\theta) \sin\theta d\theta = (A_n b^n + B_n b^{-(n+1)}) \frac{2}{2n+1}$$

Calling the integrals on the left hand side I_{an} and I_{bn} for short

$$* \quad \frac{2n+1}{2} I_{an} = A_n a^n + B_n / a^{n+1}$$

$$** \quad \frac{2n+1}{2} I_{bn} = A_n b^n + B_n / b^{n+1}$$

If we multiply * by a^{n+1} and ** by b^{n+1} and subtract

$$\frac{2n+1}{2} I_{an} a^{n+1} - \frac{2n+1}{2} I_{bn} b^{n+1} = A_n (a^{2n+1} - b^{2n+1})$$

whence

$$A_n = \frac{2n+1}{2} \left(\frac{1}{a^{2n+1} - b^{2n+1}} \right) (I_{an} a^{n+1} - I_{bn} b^{n+1})$$

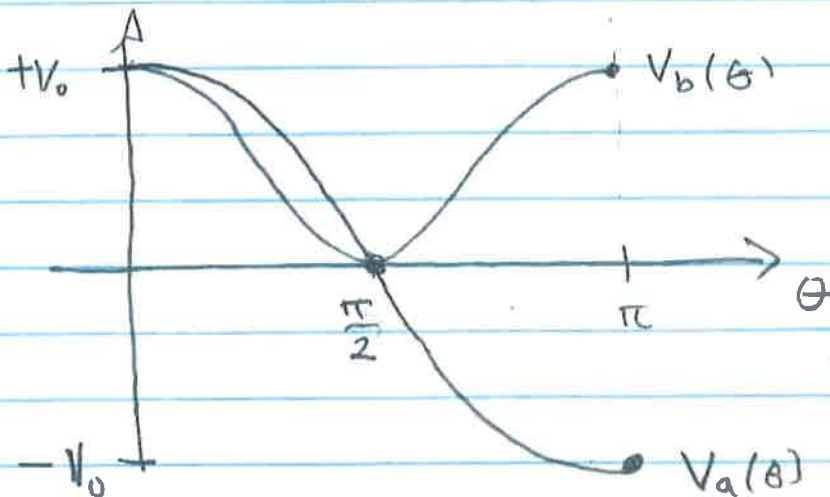
There is a similar expression for B_n :

$$B_n = \frac{2n+1}{2} \left(\frac{b^{2n+1} a^{2n+1}}{a^{2n+1} - b^{2n+1}} \right) (I_{an} / a^n - I_{bn} / b^n)$$

2-3

You are asked to do an "amusing" case.

How about $V_a(\theta) = V_0 \cos \theta$ $V_b(\theta) = V_0 \cos^2 \theta$



Clearly $V_a(\theta) = P_1(\cos \theta)$ so that $I_{a(n=1)} = \frac{2}{3} V_0$

with all other $I_{an} = 0$. Likewise $I_{b(n=1)} = \frac{2}{3} V_0$

and $I_{b(n=2)} = \frac{2}{3} \frac{2}{5} V_0$ with all other $I_{bn} = 0$ because

$$V_b(\theta) = \frac{2}{3} P_2(\cos \theta) + P_1(\cos \theta)$$

We conclude all A_n and B_n vanish except for $n=1$ and $n=2$ and, as an example of a nonvanishing term

$$A_1 = \frac{3}{2} \left(\frac{1}{a^3 - b^3} \right) \left(\frac{2}{3} a^2 - \frac{2}{3} b^2 \right) V_0$$

$$= \frac{a^2 - b^2}{a^3 - b^3} V_0.$$

similar expressions for A_2, B_1, B_2, \dots

3-1

3.

We know the sol'n of Laplace's eqn with azimuthal symmetry is

$$(*) \quad V(r, \theta) = \sum_{\ell=0}^{\infty} [A_{\ell} r^{\ell} + B_{\ell} r^{-(\ell+1)}] P_{\ell}(\cos \theta)$$

Solving outside the sphere allows $r \rightarrow \infty$ so we need $A_{\ell} = 0 \quad \forall \ell$.

The B_{ℓ} are determined by the boundary condition on the sphere's surface. Since

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{3}{2}(x^2 - 1)$$

$$\Rightarrow x^2 = \frac{2}{3} P_2(x) + P_1(x)$$

$$V(R, \theta) = V_0 \cos^2 \theta = V_0 \left[\frac{2}{3} P_2(\cos \theta) + P_1(\cos \theta) \right]$$

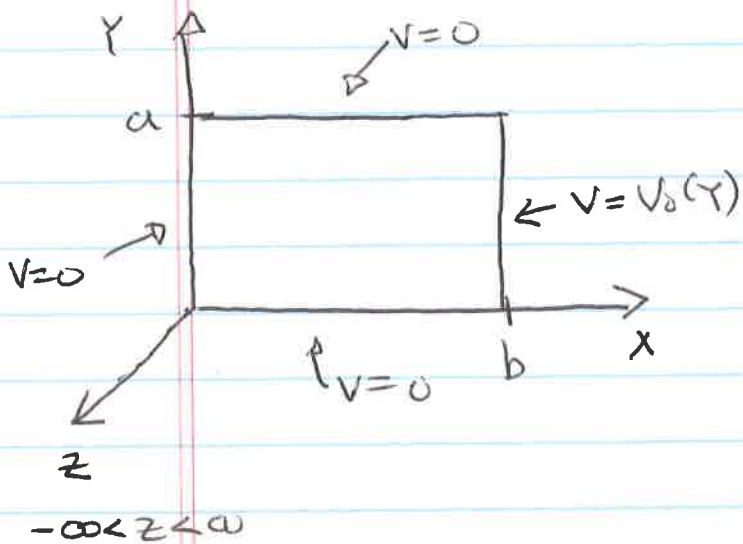
Clearly then

$$V(r, \theta) = V_0 \left(\frac{R}{r} \right)^2 P_1(\cos \theta) + V_0 \frac{2}{3} \left(\frac{R}{r} \right)^3 P_2(\cos \theta)$$

satisfies bdy conditions and is also of the general form (*). obviously we can easily do any sort of polynomial $V(R, \theta)$ by just decomposing it in $P_{\ell}(\cos \theta)$.

4-)

[4] (Griffiths 3-15)



We did this problem in class!

Separate $\nabla^2 V = 0$

$$V(x,y) = f(x)g(y)$$

$$\frac{1}{f} \frac{d^2 f}{dx^2} = \frac{1}{g} \frac{d^2 g}{dy^2} = -k^2$$

$g(y) = \sin ky; \cos ky$ \rightarrow eliminated by $V(x,0) = 0$

$$k = \frac{n\pi}{a}$$

Since

$$V(x,a) = 0$$

$f(x) = e^{+kx}; e^{-kx} \rightarrow \cosh kx; \sinh kx$

\rightarrow eliminated by $V(0,y) = 0$

$$* V(x,y) = \sum_1^{\infty} A_n \sinh \frac{n\pi x}{a} \sin \frac{n\pi y}{a}$$

set $x=b$ $V(x=b,y) = V_0(y)$

$$V_0(y) = \sum_1^{\infty} A_n \sinh \frac{n\pi b}{a} \sin \frac{n\pi y}{a}$$

Orthogonality of $\sin \frac{n\pi y}{a}$

$$\int_0^a V_0(y) \sin \frac{m\pi y}{a} dy = A_m \sinh \frac{m\pi b}{a} \frac{a}{2}$$

$$A_m = \frac{2}{a} \frac{1}{\sinh \frac{m\pi b}{a}} \int_0^a V_0(y) \sin \frac{m\pi y}{a} dy$$

combined with * is a full solution.

5 (Griffiths 3-21)

Potential of charged metal sphere in uniform E_0 .

Griffiths Example 8 concluded an uncharged

sphere had $V(r, \theta) = -E_0 \left(r - \frac{R^3}{r^2} \right) \cos \theta$

with $V=0$ on sphere surface $r=R$.

Clearly this must be part of our solution since this

is what we must get if $Q=0$. We also know

answer for $E_0=0$ is $V(r, \theta) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

By superposition principle

$$V(r, \theta) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} - E_0 \left(r - \frac{R^3}{r^2} \right) \cos \theta$$

where does this vanish? obviously at $r=\infty$

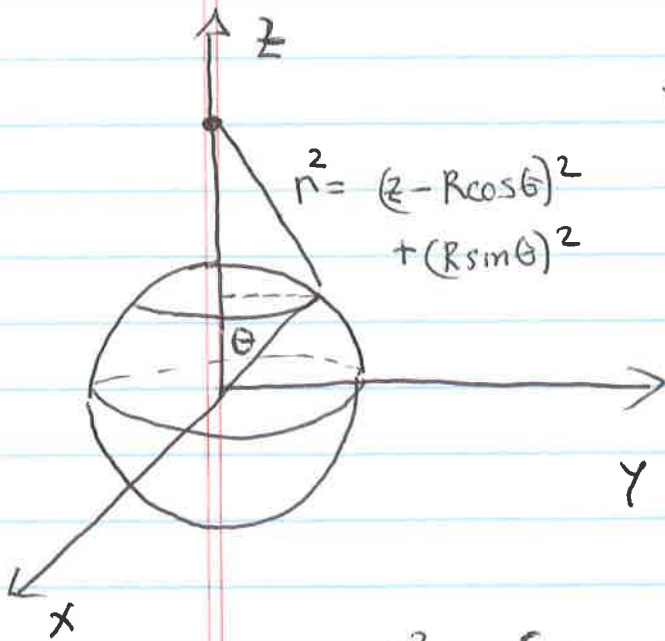
and $\theta = \pi/2$.

6-1

6 (Griffiths 3-23)

spherical shell radius R with $-\sigma_0$ for $z > 0$
and $+\sigma_0$ for $z < 0$. we can compute V along

the \hat{z} axis by explicit integration



$$V(z) = + \int_0^{\pi/2} d\theta \frac{1}{4\pi\epsilon_0} \frac{\overbrace{2\pi R \sin \theta R d\theta \sigma}^{dq}}{(z^2 - 2zR \cos \theta + R^2)^{1/2}}$$

$$- \int_{\pi/2}^{\pi} d\theta \quad \text{" (same integrand)}$$

$$V(z) = \frac{R^2 \sigma}{2\epsilon_0} \left\{ \left(z^2 - 2zR \cos \theta + R^2 \right)^{-1/2} \frac{1}{zR} \Big|_0^{\pi/2} - \left(\right) \Big|_{\pi/2}^{\pi} \right\}$$

$$= \frac{R^2 \sigma}{2\epsilon_0} \left\{ \left(\sqrt{z^2 + R^2} \frac{1}{zR} - \frac{z-R}{zR} \right) - \left(\frac{z+R}{zR} - \sqrt{z^2 + R^2} \frac{1}{zR} \right) \right\}$$

$$= \frac{R\sigma}{2z\epsilon_0} \left\{ 2\sqrt{z^2 + R^2} - z + R - z - R \right\}$$

$$= \frac{R\sigma}{\epsilon_0} \left\{ \frac{\sqrt{z^2 + R^2}}{z} - 1 \right\}$$

6-2

along the z axis $\theta = 0$ and $\begin{cases} z = r \\ \cos \theta = 1 \end{cases}$ so the
general sol'n to Laplace's Eqn

$$V(r, \theta) = \sum_{l=0}^{\infty} (A_l r^l + B_l / r^{l+1}) P_l(\cos \theta)$$

↑
vanish
since V does not
diverge at $r = \infty$

$$V(z) \equiv V(r, \theta = 0) = \sum_{l=0}^{\infty} B_l / r^{l+1} = \frac{R\sigma}{\epsilon_0} \left\{ \frac{\sqrt{r^2 + R^2} - 1}{r} \right\}$$

$$\left(1 + R^2/r^2\right)^{1/2} - 1$$

$$1 + \frac{1}{2} \frac{R^2}{r^2} - \frac{1}{8} \left(\frac{R^2}{r^2}\right)^2 + \frac{1}{16} \left(\frac{R^2}{r^2}\right)^3 - 1$$

$$B_0 = 0$$

$$B_1 = \frac{R\sigma}{\epsilon_0} \frac{1}{2} R^2$$

$$B_2 = 0 = B_4 = B_6$$

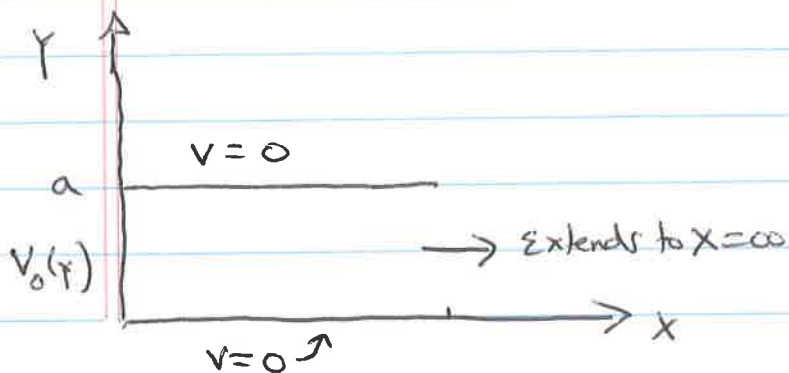
$$B_3 = \frac{R\sigma}{\epsilon_0} \left(-\frac{1}{8} R^4\right)$$

$$B_5 = \frac{R\sigma}{\epsilon_0} \left(\frac{1}{16} R^5\right)$$

$$V(r, \theta) = \frac{\sigma R}{2\epsilon_0} \left\{ \frac{R^2}{r^2} - \frac{R^4}{4r^4} + \frac{R^6}{8r^6} \dots \right\}$$

7 (Griffiths 3-14)

We are asked for the charge density $\sigma(y)$ on the strip at $x=0$ for Example 3 in text.



The general solution was

$$V(x,y) = \sum_{n=1}^{\infty} c_n e^{-n\pi x/a} \sin \frac{n\pi y}{a} \quad (\text{Eq. 30})$$

$$c_n = \frac{2}{a} \int_0^a V_0(y) \sin \frac{n\pi y}{a} dy \quad (\text{Eq. 34})$$

and for $V_0(y) = V_0$ $c_n = \begin{cases} 0 & \text{even} \\ \frac{4V_0}{n\pi} & \text{odd} \end{cases} \quad (\text{Eq. 35})$

normal component
of \vec{E}

Recall that E_n is σ/ϵ_0 just outside conductor (by Gauss' Law)

$$E_n = - \left. \frac{\partial V(x,y)}{\partial x} \right|_{x=0} = \sum_{n=1}^{\infty} c_n \frac{n\pi}{a} e^{-n\pi x/a} \sin \frac{n\pi y}{a} \Big|_{x=0}$$

$$E_n = \sum_{n=1}^{\infty} c_n \frac{n\pi}{a} \sin \frac{n\pi y}{a} \quad \leftarrow \text{general } V_0(y)$$

7-2

$$\text{For } V_0(y) = V_0 \quad c_n = \begin{cases} 0 & n \text{ even} \\ \frac{4V_0}{n\pi} & n \text{ odd} \end{cases}$$

and hence using $\sigma = \epsilon_0 E_n$

$$\sigma(y) = \sum_{n \text{ odd}} \frac{4V_0 \epsilon_0}{n\pi} \frac{n\pi}{a} \sin \frac{n\pi y}{a}$$

$$\sigma(y) = \sum_{n \text{ odd}} \frac{4V_0 \epsilon_0}{a} \sin \frac{n\pi y}{a}$$

Try sketching

Does the sum
converge?!

