PHYSICS 110A, WINTER 2017
ELECTRICITY AND MAGNETISM

Assignment Five, Due Friday, February 16, 5:00 pm.

[1.] Griffiths Problem 7, Chapter 3.

[2.] Griffiths Problem 12, Chapter 3.

[3.] Find the analytic solution of the one-dimensional Poisson equation with $\rho(x) = 12\,\epsilon_0\,x^2$,

$$-\frac{d^2\phi}{dx^2} = 12\,x^2,$$

with boundary conditions $\phi(0) = \phi(1) = 0$. What is the value of the quantity,

$$E = \int_0^1 [\ \frac{1}{2}(\frac{d\phi}{dx})^2 - 12x^2\phi\ ]dx\ ?$$

and what is its physical significance?

[4.] **(extra credit)** Solve problem 3 numerically. Use $dx = 0.1$. Make a plot containing $\phi(x)$ at a few times during the course of the iteration, and also containing the analytic solution. (I will talk about this problem on Wednesday in the "problem session" if you need guidance.)

[5.] **(extra, extra credit)** Solve Laplace's equation $\nabla^2\phi = 0$ for the potential $\phi(x, y)$ for the square region in the figure, with the boundary conditions shown. Use the iterative method discussed in class, whose form in $d = 2$ (with $\rho = 0$) is,

$$\phi(i, j) = \frac{1}{4}[\phi(i+1, j) + \phi(i-1, j) + \phi(i, j-1) + \phi(i, j+1)]\ .$$

Use $dx = 0.1$ and $a = 1$. Compare your solution with the analytic one obtained in class by plotting the two results for $\phi(x, y = a/2),\ 0 < x < a$ on the same graph. Make a second graph comparing the analytic and numeric solutions for a vertical cut, $\phi(x = a/2, y),\ 0 < y < a$.