PHYSICS 110A, WINTER 2017
ELECTRICITY AND MAGNETISM

Assignment Two, Due Friday, January 19, 5:00 pm.

[**1.**] Compute the divergence of the vector field

$$\mathbf{v} = C\,\mathbf{r}$$

In a week or so we will prove this is the electric field inside a sphere with uniform charge density ρ, if we define $C = 4\pi k\rho/3$. (You probably saw this problem in lower division E&M.) Using that statement, discuss the consistency of your result with the Maxwell Equation $\nabla \cdot \mathbf{E} = 4\pi k\rho$ which tells us the divergence of $\mathbf{E}$ measures the charge density.

[**2.**] Compute the curl of the vector field

$$\mathbf{v} = C\,(x\,\hat{\mathbf{y}} - y\,\hat{\mathbf{x}})$$

In Physics 110B you will prove this is the magnetic field inside a long straight wire with uniform current density J, if we define $C = \mu_0 J/2$. (You probably saw this problem in lower division E&M.) Using that statement, discuss the consistency of your result with the Maxwell Equation $\nabla \times \mathbf{B} = \mu_0 J$ which tells us the curl of $\mathbf{B}$ measures the current density.

[**3.**] Griffiths Problem 54, Chapter 1.

[**4.**] Griffiths Problem 55, Chapter 1.

[**5.**] Griffiths Problem 56, Chapter 1.

[**6.**] Griffiths Problem 57, Chapter 1.

[**7.**] Griffiths Problem 61, Chapter 1.

[**8.**] It is straightforward to compute the gradient of a given scalar function. In this problem we attempt the reverse process. Verify the force

$$\mathbf{F} = (3x^2yz - 3y)\hat{\mathbf{x}} + (x^3z - 3x)\hat{\mathbf{y}} + (x^3y + 2z)\hat{\mathbf{z}}$$

is conservative, and find a potential ϕ such that $\mathbf{F} = -\nabla\phi$.

[**9.**] It is straightforward to compute the curl of a given vector function. In this problem we attempt the reverse process. Find the vector field $\mathbf{A}$ such that $\mathbf{v} = \nabla \times \mathbf{A}$ for

$$\mathbf{v} = (y + z)\hat{\mathbf{x}} + (x - z)\hat{\mathbf{y}} + (x^2 + y^2)\hat{\mathbf{z}}$$

[**10.**] Compute the integral of $\nabla \cdot \mathbf{v}$ over the region $x^2 + y^2 + z^2 \leq 25$ for the vector field

$$\mathbf{v} = (x^2 + y^2 + z^2)(x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}})$$