Assignment One, Due Friday, January 12, 5:00 pm.
Note: There are more simple problems with vectors than the exercises below. Depending on how well you remember this material, you might want to review some of the more basic processes of vector addition, multiplication, etc in addition to this homework.
[1.] Find the angles between (a) the space diagonals of a cube; (b) a space diagonal and an edge; (c) a space diagonal and a diagonal of a face.
[2.] Let $\mathbf{A}=2 \mathbf{x}-\mathbf{y}-\mathbf{z}, \quad \mathbf{B}=2 \mathbf{x}-3 \mathbf{y}+\mathbf{z}$, and $\mathbf{C}=\mathbf{y}+\mathbf{z}$. Evaluate $\mathbf{A} \times(\mathbf{B} \times \mathbf{C})$ and $\mathbf{B}(\mathbf{A} \cdot \mathbf{C})-\mathbf{C}(\mathbf{A} \cdot \mathbf{B})$, and show they are equal.
[3.] Let $\mathbf{A}=2 \mathbf{x}-\mathbf{y}+2 \mathbf{z}$. (a) Find the unit vector (length 1 ) in the same direction as $\mathbf{A}$. (b) Find a vector perpendicular to $\mathbf{A}$. (c) Find a unit vector perpendicular to A.
[4.] Let $\mathbf{A}=2 \mathbf{x}-3 \mathbf{y}+\mathbf{z}$. (a) If $\mathbf{A} \cdot \mathbf{B}=0$, does it follow that $\mathbf{B}=0$ ? If not, find a specific example for $\mathbf{B}$. (b) Answer the same question if $\mathbf{A} \times \mathbf{B}=0$. (c) Answer the same question if $\mathbf{A} \cdot \mathbf{B}=0$ and $\mathbf{A} \times \mathbf{B}=0$.
[5.] What is the value of $(\mathbf{A} \times \mathbf{B})^{2}+(\mathbf{A} \cdot \mathbf{B})^{2}$ ? (Remember the square of a vector is the dot product of the vector with itself.)
[6.] Prove $(\mathbf{A} \times \mathbf{B}) \cdot((\mathbf{B} \times \mathbf{C}) \times(\mathbf{C} \times \mathbf{A}))$ is the square of $\mathbf{A} \cdot(\mathbf{B} \times \mathbf{C})$.
[7.] Extra Credit (tricky!): Use the notation and identities in pages V5A-V5B of the notes to prove the result of problem [6.].

