EF1

Extra Fun with Kronecker (δ_{ij}) and Levi-Civita (ϵ_{ijk}) Symbols!

In all below, recall the Einstein summation convention, which asserts that in any expression if an index is repeated, then it is summed over.

$\boxed{1}$ Prove that $\vec{A} \cdot \vec{B} = A_i B_i$

$\boxed{2}$ Prove that if you have two matrices P and Q

$$\underbrace{(PQ)_{ij}}_{\text{ij component of the product of P and Q}} = P_{ik} Q_{kj}$$

← this is useful if you ever write a program to multiply matrices!

(Exercises $\boxed{1}$ $\boxed{2}$ involve only the Einstein summation convention, not δ_{ij} or ϵ_{ijk}.)

EF2

3. Prove $(\vec{A} \times \vec{B})_i = \epsilon_{ijk} A_j B_k$

NOTE: Here we use 1, 2, 3 for x, y, z

4 Prove $\delta_{ii} = 3$

5 Prove $\delta_{ij} \epsilon_{ijk} = 0$

6 Prove $\epsilon_{ipq} \epsilon_{jpq} = 2\delta_{ij}$

7 Prove $\epsilon_{ijk} \epsilon_{ijk} = 6$

8 Prove $\epsilon_{ijk} \epsilon_{pqk} = (\delta_{ip} \delta_{jq} - \delta_{iq} \delta_{jp})$

9 Using ϵ_{ijk} notation, prove $\bar{A} \cdot (\bar{A} \times \bar{B}) = 0$

10 Using ϵ_{ijk} notation, prove $\bar{\nabla} \cdot (\bar{\nabla} \times \bar{A}) = 0$

11 " " " " $\bar{\nabla} \times \bar{\nabla} f = 0$

12 Derive $\bar{A} \times (\bar{B} \times \bar{C}) = \vec{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B})$

using ϵ_{ijk} notation.