PHYSICS 104B, WINTER 2010
ASSIGNMENT SIX

Due Tuesday, March 16.

Your homework solution should include a hardcopy of the code, answers to the questions, and the indicated figures. Commenting your code is strongly encouraged.

[**1.**] A system has energy levels $E(n) = \omega(n+\frac{1}{2})$, where $n = 0, 1, 2, ...$ What is $\langle E \rangle$? Name a physical system which has such a spectrum. Suppose now that $E(n) = \omega(n+\frac{1}{2}) + \lambda n^2$. Write a monte carlo program to compute $\langle E \rangle$. Do you get the correct answer at $\lambda = 0$? Comment: You need to decide how to suggest a new level n' from the current level n (after which you will do Metropolis to accept/reject). One possibility is to choose $n' = n \pm 1$, selecting the two choices with probability 0.5 each. However, you need to be very careful how you deal with the case $n = 0$ when there is no $n - 1$. To figure out the right way to deal with the special case you need to think about how to ensure detailed balance.
Extra Credit: Choose $n' = n \pm 1$, selecting $n' = n + 1$ with probability 0.7 and $n' = n - 1$ with probability 0.3 and then do Metropolis. Show you get the wrong answer and explain what went wrong with the proof that you should get the correct Boltzmann distribution when doing Monte Carlo.

[**2.**] Write a code to simulate the $d = 1$ dimensional Ising model. Set $J = 1$ and $k_B = 1$. Plot the average energy $\langle E \rangle$ and the specific heat C versus T. Get C from energy fluctuations. Simulate lattice sizes $L = 16$ and $L = 24$.

[**3.**] Redo problem [2] for dimension $d = 2$. Simulate lattice sizes $N = L$xL with $L = 16$ and $L = 24$. Show that you get a peak in $C(T)$ which is fairly close to the known position of the phase transition, $T_c = 2.236$.