PHYSICS 104B, WINTER 2010
ASSIGNMENT FIVE

Due Tuesday, March 2.

Your homework solution should include a hardcopy of the code, answers to the questions, and the indicated figures. Commenting your code is strongly encouraged.

[**1.**] Consider the coupled mass and spring system (the "one dimensional solid") discussed in class, with all masses $m_l = 1.0$ and springs $g = 2.0$ connecting masses to their near neighbors. Use periodic boundary conditions.

- Use the eigenvalue subroutine to compute the eigenvalues and eigenvectors of the "dynamical matrix" for dimension $N = 12$. Verify they agree with the analytic formula in class. Check the eigenvectors of largest and smallest eigenvalue are correct.
- Now do dimension $N = 128$. Calculate the "participation ratio" P for all the eigenvectors and show the normal modes are all extended. That is, P is within a factor of 2-3 of N.
- For the rest of this problem, continue with $N = 128$. Take several eigenvectors and plot the square of the components as a function of distance along the chain.
- Now introduce a "defect" by setting $g = 4$ for one of the springs. Use the eigenvalue subroutine to compute the eigenvalues. Do you notice a 'lonely' eigenvalue split way off from the rest? What is its numerical value? How are the rest of the eigenvalues distributed?
- Calculate the "participation ratio" for all the eigenvectors. Show that one of the normal modes is localized. That is, P is much smaller than N.
- Identify the eigenvector associated with the defect mode and plot the square of its components as a function of distance along the chain. Also plot its components without squaring. What do you notice?

[**2.**] Fill up a matrix of dimension $N = 1024$ with random numbers x with $x = \pm 1/2\sqrt{N}$ where the plus sign is taken half the time and the minus sign the other half the time. Make the matrix symmetric, $M_{ij} = M_{ji}$. Diagonalize the matrix. Do this for 10 matrices and bin the eigenvalues in bins of size 0.02. Plot the result and come up with a conjecture about the eigenvalue distribution.

[**3.**] Ten thousand visitors go to Las Vegas and play a game where they bet one dollar each time, with the probability of winning a dollar $a = 0.44$ and the probability of losing a dollar is $b = 1 - a = 0.56$. They each start with \$40, and play 400 times. How many leave Las Vegas busted? What is the average amount they leave with?
Hint: Convince yourself that you can implement the probabilities in the game by throwing a random number $0 < r < 1$ and adding a dollar to the stash when $r < a$ and subtracting a dollar when $r > a$.

[**4.**] Extra Credit] Fill up a matrix of dimension $N = 1024$ with random numbers x with $x = \pm 1/\sqrt{N}$ where the plus sign is taken half the time and the minus sign the other half the time. (Notice the difference in factor of 2 in the denominator compared to problem 2. This just scales all the eigenvalues in a way which is more consistent with problem 2.) Unlike problem 2, choose M_{ij} and M_{ji} independently, so that M is not symmetric. The eigenvalues will now be complex. Plot them in the complex plane. (Just put a little dot down for each eigenvalue, using the real part as the x component and the imaginary part as the y component.) Make a conjecture about the distribution. You will need to find a matrix diagonalizer that works for non-symmetric matrices. The one I gave you will **not**.