

Last week I enunciated some of the postulates
of quantum mechanics

(1) observable \rightarrow Hermitian operator

\nearrow
real eigenvalues!

\nearrow
sort of like a matrix
we will clarify distinction later

(2) Eigenvalues: possible results of experiment

(3) $[\hat{x}, \hat{p}] = i\hbar$

\rightarrow NEW: state of quantum mechanical system

is represented by a ^{normalized} vector $\vec{\psi}$. The probability of measuring λ_i

is given by $p_i = |\vec{v}_i^\dagger \vec{\psi}|^2$

Comment $\vec{\psi} = \sum_j a_j \vec{v}_j$ since $\{\vec{v}_i\}$ complete (we did not prove this)

$$\vec{v}_i^\dagger \vec{\psi} = \sum_j a_j \underbrace{\vec{v}_i^\dagger \vec{v}_j}_{\delta_{ij}} = a_i$$

$$\text{so } p_i = |a_i|^2$$

$$\sum p_i = 1 \Leftrightarrow \sum |a_i|^2 = 1$$

\Leftrightarrow normalized

QM2

Example $S_x = \begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix}$

Matrix corresponding to measurement of x component of spin (angular momentum) of spin $1/2$ particle like an electron

$\psi = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (properly normalized!)

Eigenvalues of S_x $\begin{pmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{pmatrix}$ $\lambda^2 - (\hbar/2)^2 = 0$
 $\lambda = \pm \hbar/2$

Eigenvectors $+\hbar/2$ $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$-\hbar/2$ $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$\frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = q_1 \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} q_2$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

Method #1 : Invert

$$\begin{pmatrix} 1 & 1 & | & 1 & 0 \\ 1 & -1 & | & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & | & 1 & 0 \\ 0 & -2 & | & -1 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & | & 1/2 & 1/2 \\ 0 & 1 & | & 1/2 & -1/2 \end{pmatrix}$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{\sqrt{10}}{10} \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Q M3

$$a_1 = 3\sqrt{10}/10 \quad (3)$$

$$a_2 = -\sqrt{10}/10$$

$$P_1 = \text{prob. of measuring } +\hbar/2 \text{ for } S_x = a_1^2 = 9/10$$

$$P_2 = \text{prob. of measuring } -\hbar/2 \text{ for } S_x = a_2^2 = 1/10$$

$$v_1^+ \psi$$

Method #2

$$a_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{1}{\sqrt{10}} \cdot 3 = \frac{3\sqrt{10}}{10}$$

$$a_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

QM 4

So given the current state of a QM system specified by vector ψ , and given a measurement we want to do \Rightarrow matrix with eigenvalues λ_i and eigenvectors v_i

we know results

"measurement process"

This is so obvious in classical mechanics

we don't bother even discussing it. If we

know "current state" — the position x and velocity v

then measuring position, velocity yields $x, v!$

Q What's missing?

In CM $F = ma \Rightarrow x(t), v(t)$

In QM ?? $\psi(t) = e^{-i\mathcal{H}t} \psi(0)!$