

EX 4 - 1

$$\begin{aligned} & \int_0^\infty \frac{x^2}{(x^2+4)(x^2+9)} dx \\ &= \frac{1}{2} \int_{-\infty}^\infty \frac{x^2}{(x^2+4)(x^2+9)} dx \end{aligned}$$

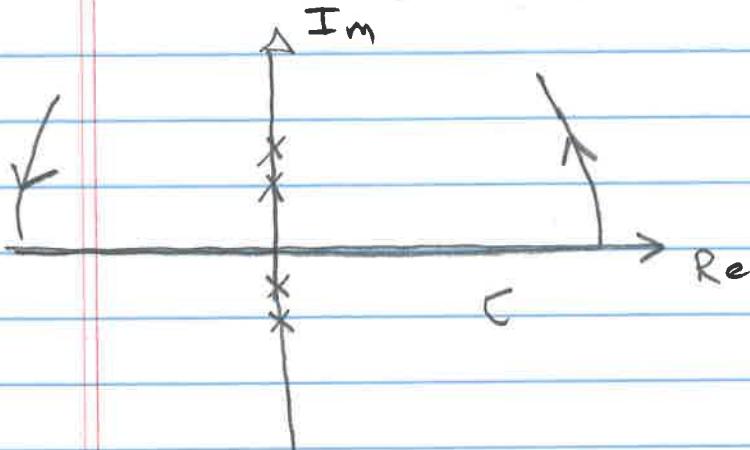
"Usual" argument

Because

$$x = z$$

along real axis and

integrand vanishes on additional semicircle



Integrand has poles at

$$z = \pm 2i$$

$$\pm 3i$$

$$f(z) = \frac{z^2}{(z^2+4)(z^2+9)} = \frac{z^2}{(z+2i)(z-2i)(z+3i)(z-3i)}$$

$$(z - 2i)f(z) = \frac{z^2}{(z+2i)(z+3i)(z-3i)}$$

$$\text{evaluate at } z = 2i \rightarrow -\frac{4}{(4i)(5i)(-i)} = -\frac{1}{5i} = \frac{c}{5}$$

$$\text{Likewise for pole at } z = 3i \text{ get } -\frac{9}{(5i)(i)(6i)} = -\frac{9i}{30} = \frac{-3i}{10}$$

Ex 4-2

Thus integral is

$$\frac{1}{2} 2\pi i \left(\frac{i}{5} - \frac{3i}{10} \right) = \frac{-\pi i}{10} (2i - 3i) = \frac{\pi}{10}$$

Can check this numerically

Useful to compute analytically the large x correction

$$\int_{x_{\max}}^{\infty} \frac{x^2 dx}{(x^2 + 4)(x^2 + 9)}$$

$$= \int_{x_{\max}}^{\infty} \frac{dx}{x^2 (1 + 4/x^2)(1 + 9/x^2)}$$

$$= \int_{x_{\max}}^{\infty} \frac{dx}{x^2} \left\{ 1 - \frac{4}{x^2} \right\} \left\{ 1 - \frac{9}{x^2} \right\}$$

$$= \int_{x_{\max}}^{\infty} \frac{dx}{x^2} \left\{ 1 - \frac{13}{x^2} \right\} = \frac{1}{x_{\max}} - \frac{13}{3x_{\max}^3}$$

N dx integral integral + correct.

1000 0.1 0.30416 0.314159

$\sqrt{\pi/10}$?

Ex - 3

```
#include <stdio.h>
#include <math.h>

int main()
{
    int i,N;
    double integral=0.0,dx,x;

    printf(" \n Enter N,dx ");
    scanf("%i %lf",&N,&dx);

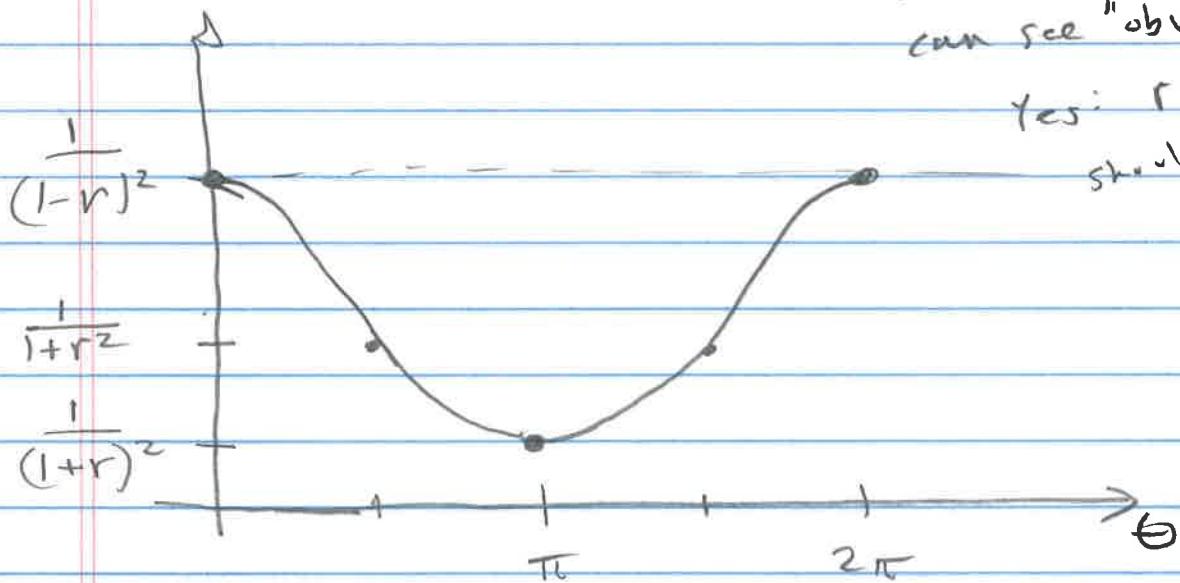
    for (i=0;i<N;i++)
    {
        x=dx*i;
        integral = integral + x*x / ( ( x*x + 4.0 )*( x*x + 9.0 ) );
    }
    integral=integral*dx;
    printf("\n integral is %12.8lf \n",integral);
    integral=integral+1.0/(N*dx);
    printf("\n integral plus correction is %12.8lf \n",integral);
}
```

Ex 5-1

$$\int_0^\pi \frac{d\theta}{1-2r\cos\theta+r^2} \quad 0 \leq r < 1$$

R Is there a limit you
can see "obviously"?

Yes: $r=0$
 $\theta = 0, \pi$.



Symmetric about π so $\frac{1}{2} \int_0^{2\pi} \frac{d\theta}{1-2r\cos\theta+r^2}$

so write $z = e^{i\theta}$

$$dz = i e^{i\theta} d\theta = i z dz$$

$$\frac{1}{2} \oint_C \frac{dz/i z}{1-2r(z+\frac{1}{z})+r^2} = \frac{1}{2i} \oint_C \frac{dz}{z-rz^2-r+r^2 z}$$

$$= \frac{i}{2} \oint_C \frac{dz}{rz^2-(1+r^2)z+r}$$

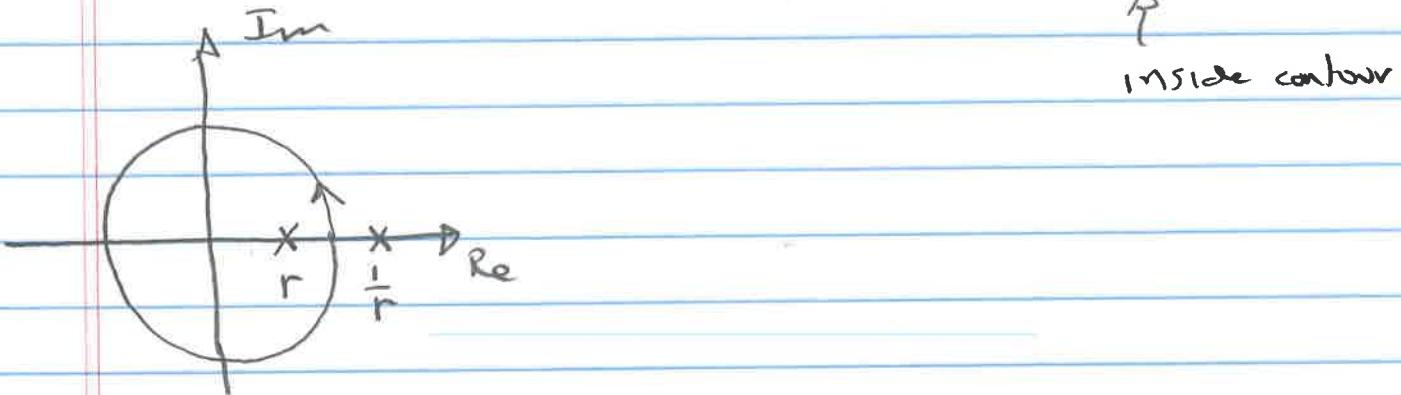
$\epsilon \times 5-2$

$$rz^2 - (1+r^2)z + r = 0$$

$$z = \left\{ (1+r^2) \pm \sqrt{(1+r^2)^2 - 4r^2} \right\} \frac{1}{2r}$$

$$= \left\{ 1+r^2 \pm \sqrt{(1-r^2)^2} \right\} \frac{1}{2r}$$

$$= \left\{ 1+r^2 \pm (1-r^2) \right\} \frac{1}{2r} = \frac{1}{r} ; r$$



$$f(z) = \frac{1}{rz^2 - (1+r^2)z + r} = \frac{1}{r(z-\frac{1}{r})(z-r)}$$

$$(z-r)f(z) = \frac{1}{r(z-\frac{1}{r})}$$

$$\text{Evaluate at } z=r \quad \frac{1}{r(r-\frac{1}{r})} = \frac{1}{r^2-1}$$

$$\text{Integral is } 2\pi i \int \frac{1}{r^2-1} = \frac{\pi i}{1-r^2} \quad \begin{matrix} \text{Correct} \\ @ r=0 \end{matrix}$$

Check numerically:

$$r=1/2 \quad \frac{\pi}{3/4} = \frac{4\pi}{3} = 4.18879$$

numerical value:

$N=100$	4.30049
200	4.24464
500	4.21113

Converging only linearly

E 5 -3

```
#include <stdio.h>
#include <math.h>

int main()
{
    int i,N;
    double integral=0.0,tpiN,theta,r;

    printf(" \n Enter N,r ");
    scanf("%i %lf",&N,&r);

    tpiN=8.0*atan(1.0)/N;

    for (i=0;i<N/2;i++)
    {
        theta=tpiN*i;
        integral = integral + 1.0 / ( 1.0 - 2.0*r*cos(theta) +r*r );
    }
    integral=integral*tpiN;
    printf("\n integral is %12.8lf \n",integral);

}
```