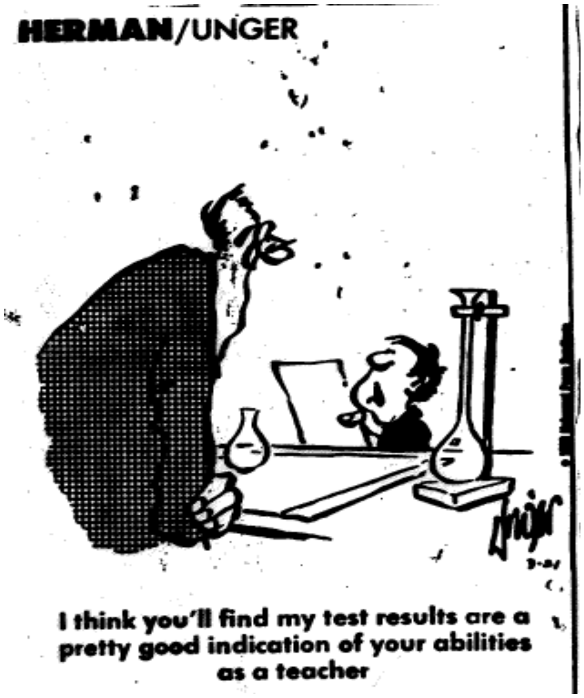

PHYSICS 104A, FALL 2015
MATHEMATICAL PHYSICS
Midterm Exam

[1.] Write down the solutions to the equation $z^6 = 2$.

[2.] In understanding the behavior of spin-1 particles in quantum mechanics, you will encounter the 3×3 matrix

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

What are the possible values you could get if you measure the x component of spin of a spin-one particle in an experiment? If your system is in the state $\vec{\psi} = (1, 0, 0)$, what are the probabilities of measuring the different possible values of S_x?

[3.] What is the definition of a projection operator?
Prove that the eigenvalues of a projection operator must be $\lambda = 0, 1$.

[4.] Use binomial theorem (left equation below) to evaluate the sum S on the right.

$$(p+q)^N = \sum_{j=0}^{N} \binom{N}{j} p^j q^{N-j} \qquad\qquad S = \sum_{j=0}^{N} \binom{N}{j} j\, p^j q^{N-j}$$

[5.] Two equal masses m are connected to each other by a spring of force constant k. One of the masses is also connected to a "wall" (ie a completely stationary object) by a spring of force constant $2k$.
(a) Write down the equations of motion $F = ma$ for the two masses.
(b) Assume a solution $x_l = v_l\, e^{i\omega t}$ for $l = 1, 2$. What equations do v_1 and v_2 obey?
(c) Compute the normal mode frequencies.
(d) Compute the normal mode vectors.
(e) In previous problems we have always found $\omega = 0$ as one of the frequencies. Did that happen here? Give a physical argument why still having $\omega = 0$ makes sense, or why you do not expect $\omega = 0$, depending on your answer.