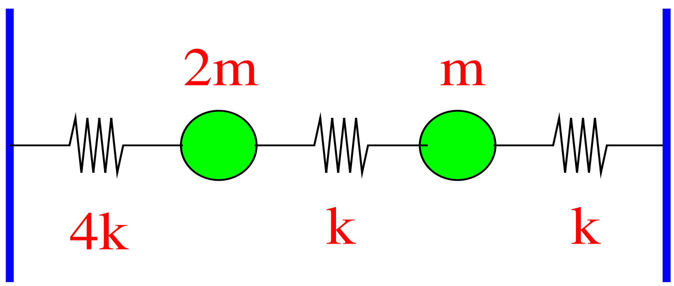
PHYSICS 104A, FALL 2018
MATHEMATICAL PHYSICS

Assignment For a Smokey Week, Due Wednesday, November 28, 5:00 pm.

[1.] (Boas Chapter 2, Section 5, problem 62).
Describe the set of points in the complex plane satisfying $|z+1|+|z-1|=8$. Doing this problem will review some analytic geometry which will be useful when you do Kepler orbits in Physics 105.

[2.] (Boas Chapter 2, Section 16, problem 3.)
Describe the motion given by $z=(1+i)\,e^{it}$.

[3.] (Boas Chapter 3, Section 12, problem 14.)
Two masses m and $2m$ are connected as shown in the Figure. Find the normal modes. Are the normal mode vectors perpendicular? If not, explain why the usual symmetric matrix $\rightarrow$ perpendicular eigenvector theorem is not applicable.



[4.] (Boas Chapter 14, Section 1, problems 10 and 21.)
Find the real and imaginary parts of the following functions:
(a) $f(z)=(2z+3)\,/\,(z+2)$
(b) $f(z)=e^{iz}$.
Using the Cauchy-Riemann conditions, check to see if (b) is analytic. Is the derivative what you expect from 'normal calculus'?

[5.] (Boas Chapter 14, Section 7, problem 15.) Evaluate

$$\int_0^\infty \frac{\cos 2x}{9x^2+4}\,dx$$

[6.] In a society on a remote Polynesian Island, all families want to have exactly two boys. When they achieve that goal, they stop having kids. So possible families are:

`BB    BGB    BGGB    BGGGB    GBB    GBGB    GBGGB    GBGGGB    GGBB    [...]`

At each birth, the probabilities of having a boy and a girl are equal. The average number of boys in a family is obviously two, since each family has exactly two boys. What is the average number of girls? First, give a conceptual argument. Then verify it by an explicit calculation.

[7.] In the linear algebra notes, part 4, you learned how to get the normal modes for a set of N masses m which are all equal, connected by springs k which are equal (if you use periodic boundary conditions).

From the equation in the notes, show that ω is linearly proportional to q for small q. This observation is the origin of the name "phonons" for lattice vibrations of a solid: the relation between energy ω and momentum q is the same as for *photons*, $\omega = c\,q$ where c is the speed of light. What is the speed of sound, the constant of proportionality between ω and q for our mass-spring system, in terms of k and m?

[8.] **Extra credit, very super hard!** Consider a set of N masses m which are still all equal, but connected by springs which *alternate* in value: $k, 2k, k, 2k, k, \ldots$. Compute the normal modes, and hence the relation between ω and q. You will find (**it's hard!**) that there are actually two types of modes. One is called "acoustic phonons" and the other is called "optical phonons". The algebra in this problem is also a simple model of band gaps for electrons moving in a solid like polyacetylene (a dimerized chain of carbon atoms) where the distances between adjacent carbon atoms *alternates* from short to long, and hence the ability of electrons to 'hop' from carbon to carbon alternates between easy and hard. So it's a nice problem to master.

Comment on [7]: The linear relation between ω and q for phonons is only true for small q. For larger q the energy 'bends over' and reaches a maximum. This is because the atoms in a solid have a spacing d and no vibrations with wavelength shorter than d are possible (minimum in wavelength implies maximum in energy since E is proportional to $1/\lambda$). The fact that there are maximum energies associated with the 'discrete nature of space' in a solid, sometimes makes solid state physics problems more simple than their high energy counterparts. In high energy physics, there are many awful 'ultraviolet divergences' where things blow up because the energy can get bigger and bigger without bound. One way high energy people deal with that is to introduce artificial discretizations of space!