# PHYSICS 104A, FALL 2015
# MATHEMATICAL PHYSICS

**Assignment Eight, Due Tuesday, December 8, 5:00 pm.**

Problems 2-4 are the same as the last three problems of Assignment 7, except notice the additional final part of problem 4.

[1.] A quantum mechanical particle has initial wave function $\psi(x,0) = ne^{-a|x|/2}$. What value of $n$ is needed for normalization? What is the distribution of momentum $c(k)$? Check that $c(k)$ is normalized. Compute $\Delta x \Delta p$. Does it have the minimum possible value $\hbar/2$ as did the Gaussian wave function discussed in class?

[2.] The temperature of a thin metallic plate is given by $T(x,y) = xy - y$. Sketch the isothermal curves corresponding to $T = 0, 1, 2$. In what direction is the temperature changing most rapidly at the point $(x,y) = (1,1)$? What is the directional derivative of $T$ at $(1,1)$ along the direction of $3\,\hat{i} + 4\,\hat{j}$?

[3.] Compute the divergence and curl of

(**a**) $\vec{V} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ (**b**) $\vec{V} = y\,\hat{i} + z\,\hat{j} + x\,\hat{k}$

(**c**) $\vec{V} = x^2\,\hat{i} + y^2\,\hat{j} + z^2\,\hat{k}$ (**d**) $\vec{V} = x^2y\,\hat{i} + y^2x\,\hat{j} + xyz\,\hat{k}$

[4.] Are the following force fields conservative?

$$\vec{F}_1 = -y\,\hat{i} + x\,\hat{j} + z\,\hat{k}$$
$$\vec{F}_2 = +y\,\hat{i} + x\,\hat{j} + z\,\hat{k}$$

Compute their integrals for a particle moving clockwise around the circle $x^2 + y^2 = a^2$ at $z = 0$. You just computed $\int_C \vec{F} \cdot d\vec{l}$. Show that you get same answer by using Green's theorem, $\int_C \vec{F} \cdot d\vec{l} = \int_S \vec{\nabla} \mathrm{x} \vec{F} \cdot \hat{n}\, dA$.

[5.] Show that Gauss's law is satisfied for the vector field of problem 3a where the volume being integrated over is a sphere of radius $R$ centered at the origin.