PHYSICS 104A, FALL 2015 MATHEMATICAL PHYSICS

Assignment Seven C, Due Monday, December 5, 5:00 pm.

[1.] The displacement from equilibrium of a violin string of length L is given by y(x,t). The string is plucked so that its initial displacement is

$$y(x,t=0) = \frac{2h}{L}x$$

$$0 < x < \frac{L}{2}$$

$$y(x,t=0) = \frac{2h}{L}(L-x)$$

$$\frac{L}{2} < x < L$$

It is released from rest so that the initial velocity $\partial y(x,t)/\partial t\big|_{t=0}=0$. Compute y(x,t) for later times. (A considerable amount of the mathematics will be similar to problem number one.)

[2.] Solve Laplace's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

inside the rectangular box 0 < x < L and 0 < y < H. The edges of the box along the axes are grounded: V(x=0,y)=0 and V(x,y=0)=0, as is the potential along the top edge: V(x,y=H)=0. Along the right edge x=L the potential rises linearly from V(L,y=0)=0 to $V(L,y=H)=V_0$. That is, $V(L,y)=V_0y/H$.

[3.] A quantum mechanical particle has initial wave function $\psi(x,0) = ne^{-a|x|/2}$. What value of n is needed for normalization? What is the distribution of momentum c(k)? Check that c(k) is normalized. Compute $\Delta x \Delta p$. Does it have the minimum possible value $\hbar/2$ as did the Gaussian wave function discussed in class?

Physics 104A Assignment 7

The wave 624 12 2x3 - 12 2x3 = 0

where y(xt) is he displacement from equilibrium.

For a vibrating string y(D, t) = y(L, t) = 0.

If we reparale variables y(xt) = f(x)g(t)

we see hat $f''(x) = -k^2 f(x)$

 $g''(t) = -v^2k^2g(t)$

So hat fl=) = sinkx ; coskx

ght) = sinut; cosut where w= vk

The boundary enditons eliminate the coskx solin

and require k= nt/6 so hat

 $y(x,t) = \sum_{i=1}^{20} \sin \frac{n\pi x}{L} \left\{ q_{i}\cos \frac{n\pi x}{L} + b_{i}\sin \frac{n\pi x}{L} \right\}$

and $\frac{34}{5t}(x,t) = \frac{5}{2} \left(\frac{5m}{L} \right) \left(\frac{n\pi v}{L} \right) \left(\frac{n\pi v}{$

In this particular problem we are told

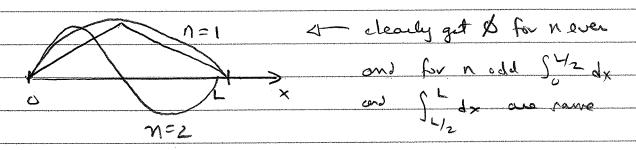
$$50 y(xt) = \sum_{i=1}^{\infty} q_{i} \sin \frac{n\pi x}{L} \cos \frac{m_{i}vt}{L}$$

we also know

$$y(x,0) = \sum_{i} q_{i} \sin \frac{1}{L} = \frac{2h_{i}(x)}{2h_{i}(L-x)} \frac{2h_{i}(x)}{2h_{i}(L-x)}$$

The fund step is getting the an Multiply by SIM I and integrate

or moderality: \frac{7}{2} dW = \frac{7}{7} \left(x''o) \frac{2}{7} \left(x''o) \frac{7}{7} \left(x''o) \frac(x''o) \frac{7}{7} \left(x''o) \frac{7}{7} \left(x''o) \frac{7}{7



$$d_{m} = \frac{4}{L} \frac{2h}{L} \int_{0}^{4} \frac{2h}{x} \sin \frac{\pi x}{L} dx$$

$$A_{M} = \frac{8h}{L^{2}} \frac{L}{m\pi} \left\{ -\frac{L}{2} \cos \frac{m\pi\tau}{2} + 0 + \frac{L}{m\pi\tau} \sin \frac{m\pi\tau}{2} \right\}$$

$$= \frac{8h}{m\pi\tau} \left\{ -\frac{L}{2} \cos \frac{m\pi\tau}{2} + \frac{L}{m\pi\tau} \sin \frac{m\pi\tau}{2} \right\}$$

recall m is old so $\cos \frac{m\pi}{2} = 0$ and $\sin \frac{m\pi}{2} = (-1)^{\frac{m}{2}}$

$$q_{m} = \frac{8h}{m^{2}\pi^{2}}(-1)^{\frac{M-1}{2}}$$

$$y(x,t) = \sum_{m=0}^{\infty} q_{m} \sin \frac{m\pi x}{L} \cos \frac{m\pi vt}{L}$$

To a void formy looking m-1/2 sometimes

we write odd integers as 2m+1 m=011,2,,.

and tre-

$$q_{m} = \frac{8h}{(2m+1)^{2} \pi^{2}} (-1)^{m}$$

$$y(x,t) = \sum_{m=0,1/2} q_m \sin \frac{(2m+1)\pi x}{L} \cos \frac{(2m+1)\pi x}{L}$$

of course that makes arguments of sine, cosing more complicated.

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N=1

2-2 To determine 9 set X=L VOYA = = ansm H cosh mil Integrate Sin mtry / H on both rides, use orthogonality It vor sin may by = H am cash math We did this integral in problem #1, .. , but .. again TO SYSIAMITY IN = VO H J-Y COSMITY H + MESSMITY LY) = Vo JH COS MTE + H SINMTEY H } = <u>vo</u> H { (-1) m+1 } am = 210 (-1) m+1 // cosh mar L/H V(x,y) = Zi am sin mtey cosh max + Zbm Sin MTX cosh MTY bm = 206 (-1) M+1 1/ cosh m+1 H/ Just exchange

LEON

$$\psi(x,0) = ne^{-a|x|/2}$$

Normalization condition

$$1 = \int_{-\infty}^{\infty} |\Psi(x,o)|^2 dx = \eta^2 \int_{-\infty}^{\infty} e^{-\alpha |x|} dx$$

$$=2n^2\int_0^\infty e^{-\alpha x}dx=2n^2e^{-\alpha x}\Big|_0^\infty=\frac{2n^2}{\alpha}$$

$$\therefore n = \sqrt{\frac{9}{2}}$$

Momentum prob dist, s former transform

$$= \eta \int e^{\alpha x/2} e^{ikx} dx + 1 \int e^{-\alpha x/2} e^{ikx} dx$$

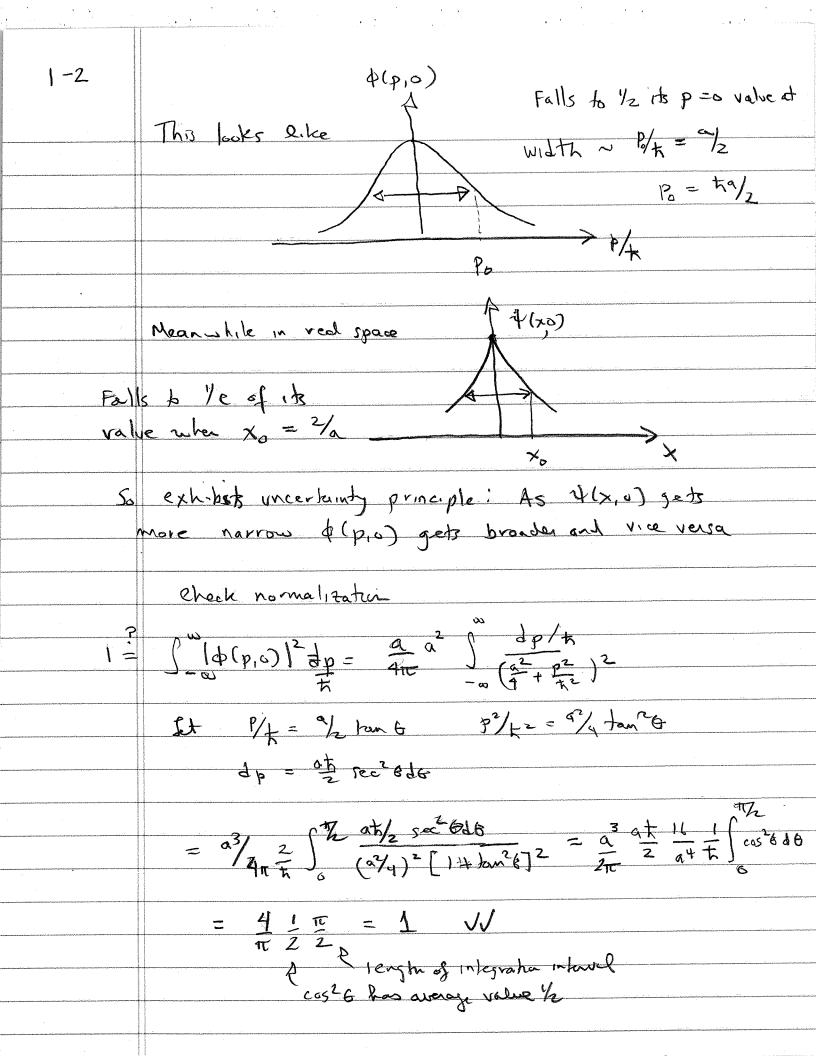
$$= \eta \int e^{\alpha x/2} e^{ikx} dx + 1 \int e^{-\alpha x/2} e^{ikx} dx$$

$$= \frac{n}{\sqrt{2\pi}} \frac{(-4/2 + ik) \times |0|}{\sqrt{2\pi}} + \frac{(-4/2 + ik) \times |0|}{\sqrt{2\pi}} + \frac{(-4/2 + ik) \times |0|}{\sqrt{2\pi}}$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}} + \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}} + \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}$$

$$= \frac{n}{\sqrt{z_{rr}}} \frac{-4/2 + ik - 4/2 - ik}{-a^2/4 - k^2} = \frac{na}{\sqrt{z_{rr}}} \frac{(a^2/4 + k^2)}{(a^2/4 + k^2)}$$

$$\Phi(p, 0) = \sqrt{\frac{a}{4\pi}} \frac{a}{4^2 + p_{4^2}^2}$$



Check uncertainty relation

$$\Delta X^2 = \langle X^2 \rangle - \langle X \rangle^2$$

$$\langle x^2 \rangle = \eta^2 \int_{-\infty}^{\infty} x^2 e^{-a|x|} dx$$

$$=2n^{2}\left\{\frac{x^{2}e^{-ax}}{-a}\right\}+\frac{2}{a}\int_{0}^{\infty}xe^{-ax}dx$$

$$= \frac{4m^2}{a} \left\{ \frac{xe^{-\alpha x}}{-a} \right\}^{\infty} + \frac{1}{a} \int_{0}^{\infty} e^{-\alpha x} dx$$

$$= \frac{4n^2}{a^2} = \frac{e^{-ax}}{a^3} = \frac{4n^2}{a^3} = \frac{2}{a^3} = \frac{2}{a^2}$$

$$\Delta p^2 = \langle p^2 \rangle - \langle p \rangle^2$$

$$\langle p^2 \rangle = \int_{-\omega}^{\omega} |\Phi(p, \omega)|^2 p^2 dp = \frac{a^2 q^2}{4\pi t^2} \int_{-\omega}^{\infty} \frac{p^2}{4} dp$$

Some try substitution is before ...

$$= \frac{a^3}{2\pi h} \frac{ah}{2} \left(\frac{ha}{2}\right)^2 \int_{0}^{\pi/2} \sin^2 \theta d\theta$$

1/2 1/2 as before

Thurs
$$\langle p^2 \rangle = \frac{4}{\pi} \frac{t^2 a^2}{4} + \frac{t^2 a^2}{4}$$

In summary
$$\Delta x = \sqrt{z/a}$$

$$\Delta p = \frac{\hbar a}{2}$$

$$\Delta \times \Delta p = h/\sqrt{2}$$
 I this is longer than

the minimum possible

Value h/z ,