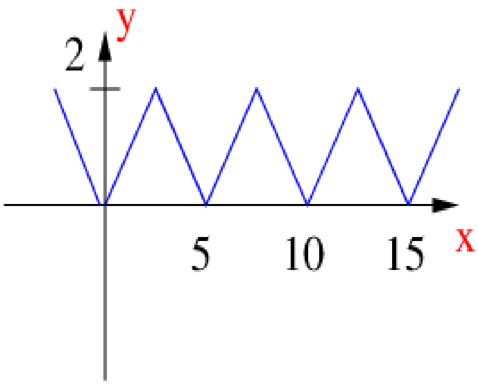
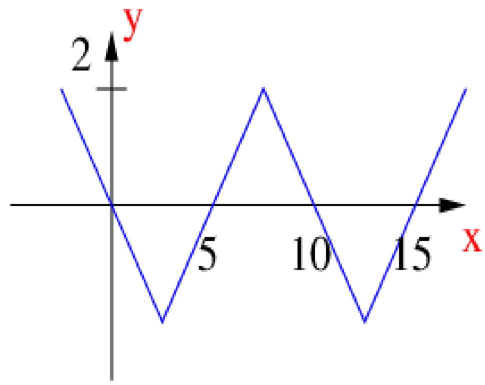
PHYSICS 104A, FALL 2018
MATHEMATICAL PHYSICS

Assignment Seven, Due Monday, December 10, 5:00 pm.

[1.] Do the Fourier decompositions of the periodic functions shown in the figures.





[2.] 'Rectification' of $V(t)$ preserves unchanged any positive voltages, but sets any negative voltage to zero. What is the harmonic content resulting from rectification of the voltage from a wall socket, $V_{\rm ac}(t) = 120 \sin(120\pi t)$? That is, what frequencies does it contain and in what proportions?

[3.] A quantum mechanical particle is confined to a box $0 < x < L$ and has initial wave function $\psi(x, t = 0) = A$ for $L/4 < x < 3L/4$. That is, the particle starts out in the central half of the box. Compute $\psi(x, t)$.

[4.] The displacement from equilibrium of a violin string of length L is given by $y(x, t)$. The string is plucked so that its initial displacement is

$$\begin{aligned} y(x, t = 0) &= \frac{2h}{L}\, x & 0 < x < \frac{L}{2} \\ y(x, t = 0) &= \frac{2h}{L}\, (L - x) & \frac{L}{2} < x < L \end{aligned}$$

It is released from rest so that the initial velocity $\partial y(x,t)/\partial t\big|_{t=0} = 0$. Compute $y(x, t)$ for later times. (A considerable amount of the mathematics will be similar to problem number one.)

[5.] Solve Laplace's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

inside the rectangular box $0 < x < L$ and $0 < y < H$. The edges of the box along the axes are grounded: $V(x = 0, y) = 0$ and $V(x, y = 0) = 0$, as is the potential along the top edge: $V(x, y = H) = 0$. Along the right edge $x = L$ the potential rises linearly from $V(L, y = 0) = 0$ to $V(L, y = H) = V_0$. That is, $V(L, y) = V_0 y/H$.

[6.] Cancelled!

[7.]
(a) Use the generating function to compute the first four Legendre polynomials.
(b) Verify by explicit integration that $P_2(x)$ is perpendicular to $P_3(x)$ and $P_4(x)$.
(c) Verify the normalization condition

$$\int_{-1}^{1} \left(P_l(x) \right)^2 dx = \frac{2}{2l+1}$$

for $l = 2$.
(d) Verify the recursion relation

$$l\, P_l(x) = (2l-1)\, x\, P_{l-1}(x) - (l-1)\, P_{l-2}(x)$$

is correct for $l = 3$.