PHYSICS 104A, FALL 2015
MATHEMATICAL PHYSICS

Assignment Seven, Due Tuesday, November 24, 5:00 pm.

[1.] The displacement from equilibrium of a violin string of length L is given by $y(x,t)$. The string is plucked so that its initial displacement is

$$y(x,t=0) = \frac{2h}{L}\,x \qquad\qquad 0 < x < \frac{L}{2}$$
$$y(x,t=0) = \frac{2h}{L}\,(L-x) \qquad\qquad \frac{L}{2} < x < L$$

It is released from rest so that the initial velocity $\partial y(x,t)/\partial t\big|_{t=0} = 0$. Compute $y(x,t)$ for later times.

[2.] Solve Laplace's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

inside the rectangular box $0 < x < L$ and $0 < y < H$. The edges of the box along the axes are grounded: $V(x=0,y) = 0$ and $V(x,y=0) = 0$. Along the other two edges the potential rises linearly from zero to $V(L,H) = V_0$. That is, $V(L,y) = V_0 y/H$ and $V(x,H) = V_0 x/L$. Hint: The Laplace equation is linear.

[3.] The temperature of a thin metallic plate is given by $T(x,y) = xy - y$. Sketch the isothermal curves corresponding to $T = 0, 1, 2$. In what direction is the temperature changing most rapidly at the point $(x,y) = (1,1)$? What is the directional derivative of T at $(1,1)$ along the direction of $3\,\hat{\imath} + 4\,\hat{\jmath}$?

[4.] Compute the divergence and curl of

(**a**) $\vec{V} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$ (**b**) $\vec{V} = y\,\hat{\imath} + z\,\hat{\jmath} + x\,\hat{k}$

(**c**) $\vec{V} = x^2\,\hat{\imath} + y^2\,\hat{\jmath} + z^2\,\hat{k}$ (**d**) $\vec{V} = x^2 y\,\hat{\imath} + y^2 x\,\hat{\jmath} + xyz\,\hat{k}$

[5.] Are the following force fields conservative?

$$\vec{F}_1 = -y\,\hat{\imath} + x\,\hat{\jmath} + z\,\hat{k}$$
$$\vec{F}_2 = +y\,\hat{\imath} + x\,\hat{\jmath} + z\,\hat{k}$$

Compute their integrals for a particle moving clockwise around the circle $x^2 + y^2 = a^2$ at $z = 0$.