PHYSICS 104A, FALL 2015
MATHEMATICAL PHYSICS

Assignment Six, Due Friday, November 13, 5:00 pm.

[1.] Compute the Fourier Series for the function $f(x) = 4 - x^2$ with $-2 < x < 2$ which repeats with period $L = 4$. Extra Credit: Plot the function and its Fourier series approximants ending the series at maximal $n = 1, 2, 4, 8, 16$.

[2.] An LRC circuit is driven by a voltage

$$\begin{aligned} V(t) &= -V_0 \qquad && -T < t < 0 \\ V(t) &= +V_0 \qquad && 0 < t < +T \end{aligned}$$

which is periodic with period $2T$. Find the charge on the capacitor $Q(t)$ as a function of time. You can leave your answer as an infinite sum. Also, ignore the 'transients', i.e. find the solution for $t >> 2L/R$ so that the initial values $Q(t = 0)$ and $I(t = 0)$ are irrelevant.

[3.] A quantum mechanical particle is initially confined in the left hand half of an infinite square well $0 < x < L$. That is,

$$\begin{aligned} \psi(x, t = 0) &= \sqrt{\frac{2}{L}} \qquad && 0 < x < \frac{L}{2} \\ \psi(x, t = 0) &= 0 \qquad && \frac{L}{2} < x < L \end{aligned}$$

Compute the wave function $\psi(x, t)$ at later time t. Extra Credit: Plot the Fourier series approximant for $\psi(x, t = 0)$ ending the series at maximal $n = 32, 128$. Extra Extra Credit: Plot $|\psi(x, t)|^2$ using maximal $n = 128$ for a couple of interesting t values. (The point is to watch the wave function spread out into the unoccupied region of the box.)