PHYSICS 104A, FALL 2018 MATHEMATICAL PHYSICS

Assignment Five, Due Tuesday, November 6, 5:00 pm.

[1.] In part one of this assignment you encountered the 3×3 matrices

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0 \end{pmatrix} \qquad S_z = \hbar \begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1 \end{pmatrix}$$

What are the possible values you could get if you measure the x, y or z component of spin of a spin-one particle in an experiment? If your system is in the state $|\psi\rangle = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, what are the probabilities of measuring the different possible values of S_x ?

[2.] Construct the matrix $S^2 = S_x^2 + S_y^2 + S_z^2$ for the spin of a spin-one particle. What possible values can you get if you measure the square of the spin?

[3.] Go back to the spin-1/2 matrices of problems 1-3 and similarly construct the matrix $S^2 = S_x^2 + S_y^2 + S_z^2$. What possible values can you get if you measure the square of the spin?

Note: It's a bit weird that when you measure S^2 it is not really what you would naively expect. The reason is that S_x , S_y and S_z do not commute, and so you cannot measure them at the same time. Thus you cannot really hope to get S^2 by just taking the components, squaring them, and adding.