## MATHEMATICAL PHYSICS

Assignment Five, Due Tuesday, November 6, 5:00 pm.
[1.] In part one of this assignment you encountered the $3 \times 3$ matrices

$$
S_{x}=\frac{\hbar}{\sqrt{2}}\left(\begin{array}{ccc}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0
\end{array}\right) \quad S_{y}=\frac{\hbar}{\sqrt{2}}\left(\begin{array}{ccc}
0 & -i & 0 \\
i & 0 & -i \\
0 & i & 0
\end{array}\right) \quad S_{z}=\hbar\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{array}\right)
$$

What are the possible values you could get if you measure the $x, y$ or $z$ component of spin of a spin-one particle in an experiment? If your system is in the state $|\psi\rangle=(1 / \sqrt{3}, 1 / \sqrt{3}, 1 / \sqrt{3})$, what are the probabilities of measuring the different possible values of $S_{x}$ ?
[2.] Construct the matrix $S^{2}=S_{x}^{2}+S_{y}^{2}+S_{z}^{2}$ for the spin of a spin-one particle. What possible values can you get if you measure the square of the spin?
[3.] Go back to the spin-1/2 matrices of problems 1-3 and similarly construct the matrix $S^{2}=S_{x}^{2}+S_{y}^{2}+S_{z}^{2}$. What possible values can you get if you measure the square of the spin?

Note: It's a bit weird that when you measure $S^{2}$ it is not really what you would naively expect. The reason is that $S_{x}, S_{y}$ and $S_{z}$ do not commute, and so you cannot measure them at the same time. Thus you cannot really hope to get $S^{2}$ by just taking the components, squaring them, and adding.

