PHYSICS 104A, FALL 2018
MATHEMATICAL PHYSICS

Assignment Five, Due Friday, November 2, 5:00 pm.

[1.] In understanding the behavior of spin-1/2 particles in quantum mechanics, you will encounter the 2×2 'Pauli matrices'

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Compute the inverses of σ_x, σ_y, and σ_z.

[2.] The 'commutator' of two matrices A and B is symbolized by $[A, B]$ and is defined by

$$[A, B] = AB - BA$$

Show that the 'spin' matrices

$$S_x = \frac{\hbar}{2}\sigma_x \qquad S_y = \frac{\hbar}{2}\sigma_y \qquad S_z = \frac{\hbar}{2}\sigma_z$$

obey

$$[S_x, S_y] = i\hbar S_z \qquad [S_y, S_z] = i\hbar S_x \qquad [S_z, S_x] = i\hbar S_y$$

[3.] to understand how spin wave functions evolve in time t you will need the exponentials of the Pauli matrices,

$$A = e^{-it\sigma_x} \qquad B = e^{-it\sigma_y} \qquad C = e^{-it\sigma_z}$$

Using the definition

$$e^M = I + M + \frac{1}{2}M^2 + \frac{1}{6}M^3 + \cdots$$

compute A, B and C.

[4.] In understanding the behavior of spin-1 particles in quantum mechanics, you will encounter the 3×3 matrices

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Compute the commutators $[S_x, S_y]$, $[S_y, S_z]$, and $[S_z, S_x]$ for spin-1. How do the results compare to the spin-1/2 case?