PHYSICS 104A, FALL 2015
MATHEMATICAL PHYSICS

Assignment Five, Due Tuesday, November 3, 5:00 pm.

[1.] In the "usual" basis

$$\hat{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \hat{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

a vector $\vec{v}$ has the components given below, and an operator $\mathcal{O}$ is represented by the matrix also given below.

$$\vec{v} \text{ ``}=\text{''} \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \qquad \mathcal{O} \text{ ``}=\text{''} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

What do the components of $\vec{v}$ and the matrix representing $\mathcal{O}$ become in the new basis:

$$\hat{f}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \hat{f}_2 = \begin{pmatrix} 0 \\ 0.6 \\ 0.8 \end{pmatrix} \qquad \hat{f}_3 = \begin{pmatrix} 0 \\ -0.8 \\ 0.6 \end{pmatrix}$$

You can use the formulae from class, but it might be even better to rederive them yourself so they really stick with you.

[2.] The Hamiltonian of a quantum mechanical system, and its initial wavefunction, are given by

$$H = \hbar\,\omega_0 \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \qquad \vec{\Psi}(t=0) = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$$

Here ω_0 is a number with the units of inverse time (frequency). Using the basic rule for time evolution in quantum mechanics, $\vec{\Psi}(t) = \exp\left(-iHt/\hbar\right) \vec{\Psi}(t=0)$, compute $\vec{\Psi}(t)$.

[3.] For the same quantum mechanical system of problem 2, there is also a quantity Q you want to measure. It has the matrix

$$Q = Q_0 \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

What are the probabilities to get different possible values for Q? (Note: the answers will depend on time!)