PHYSICS 104A, FALL 2016
MATHEMATICAL PHYSICS

Assignment Four, Due Saturday, October 27, 5:00 pm.

[1.] Compare the relativistic energy $E = mc^2(1 - v^2/c^2)^{-1/2}$ to the classical energy $\frac{1}{2}mv^2$ by expanding the former as a series in powers of v/c.

[2.] Write down the probabilities for n heads if you toss a coin $N = 10$ times and each *individual* toss has a probability $p = 0.6$ of being heads.

[3.] Show that the root mean square distance from the origin after N steps of a random walk in one dimension with probability $p = 1/2$ of going one step to the left and probability $q = 1/2$ of going one step to the right is equal to $\sqrt{N}$.

[4.] Suppose you flip a coin $N = 10000$ times. If the coin is fair (equal probability of heads and tails), the most likely outcome is 5000 heads and 5000 tails. But you probably would not be too surprised if you got 4997 heads and 5003 tails. Discuss at what point you *would be surprised*. Would you suspect the coin is not fair if you got 4700 heads and 5300 tails? What is a good, general rule for a reasonable deviation from $N/2$ heads, for arbitrary N?

[5] An old 'brain-twister' is the following fictional story: Long ago there was a society where every family was supposed to have at least one daughter. Parents were asked to have children until they finally have the desired girl. At that point their family was complete. So, possible familly structure (sequences of children) were:
G; **BG**; **BBG**; **BBBG**; $\cdots$ where **G**=girl and **B**=boy. Assuming that at each birth the probabilities of a boy or girl are equal (exactly one-half), compute the gender distribution in this society. Are there more boys or more girls?

[6.] When you do "mean field theory" to determine the critical temperature T_c at which a material becomes magnetic, you need to solve the transcendental equation, $m = \tanh(Jm/T)$, Here m is the magnetization and J is called the 'exchange parameter'. (J is determined by the overlab of the wave functions of electrons on adjacent atoms.) Draw pictures of $\tanh(Jm/T)$ for $T = 2.0\,J, 1.3\,J$, and $0.7\,J$. What can you say about these curves intersecting a line of slope one, that is, the nature of solutions to $m = \tanh(Jm/T)$? Use a series expansion of tanh to prove that $m = 0$ is the only solution for $T > J$, but that there are 'nonzero magnetization' solutions with $m \neq 0$ when $T < J$. The big physics conclusion is that you have computed the critical temperature! It is $T_c = J$!
Extra Credit: Show that $m(T) \sim (T - T_c)^{\frac{1}{2}}$ for T just a bit below T_c. This is not a difficult calculation, like last week's Fermi function. It is just built on carefully thinking through which quantities are small. Neverthless, the result is really important! The power $1/2$ is the "mean field critical exponent" of the magnetic phase transition!