PHYSICS 104A, FALL 2015
MATHEMATICAL PHYSICS

Assignment Four, Due Friday, October 23, 5:00 pm.

[1.] Solve for the eigenfrequencies and eigenvectors (i.e. the normal modes) of eight equal masses connected by eight springs with periodic boundary conditions. Apply the general formulae for arbitrary N to the special case $N = 8$. Draw "pictures" of the motion of the individual masses for each mode.

[2.] In understanding the behavior of spin-1 particles in quantum mechanics, you will encounter the 3×3 matrices

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

What are the possible values you could get if you measure the x, y or z component of spin of a spin-one particle in an experiment? If your system is in the state $|\psi\rangle = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, what are the probabilities of measuring the different possible values of S_x?

[3.] Construct the matrix $S^2 = S_x^2 + S_y^2 + S_z^2$ for the spin of a spin-one particle. What possible values can you get if you measure the square of the spin?

[4.] Go back to the spin-1/2 matrices of assignment two and similarly construct the matrix $S^2 = S_x^2 + S_y^2 + S_z^2$. What possible values can you get if you measure the square of the spin?

[5.] Compute the commutators $[S_x, S_y]$, $[S_y, S_z]$, and $[S_z, S_x]$. How do the results compare to the spin-1/2 case?

Note: It's a bit weird that when you measure S^2 it is not really what you would naively expect. The reason is that S_x, S_y and S_z do not commute, and so you cannot measure them at the same time. Thus you cannot really hope to get S^2 by just taking the components, squaring them, and adding.

[6.] Extra Credit: You have the matrices S^2 for $S = 1/2$ and for $S = 1$. What is the form of the matrix for the S^2 for a particle of general spin S? This same result will come up in Physics 115AB when you figure out the eigenvalues of the orbital angular momentum of a electron going around a proton in the Hydrogen atom, since the angular momentum matrices obey the same basic rules as spin matrices. You saw it here first.