PHYSICS 104A, FALL 2018
MATHEMATICAL PHYSICS

Assignment Three, Due Friday, October 19, 5:00 pm.

[1.] Let $\vec{F} = 2xy^2\,\hat{\imath} + x^2\hat{\jmath}$. Compute $\int_{\mathcal{P}} \vec{F} \cdot d\vec{r}$ from the origin to $(1,1)$ (a) along the path $y = x^2$; and (b) along the path $y = x$. Comment on your results. Are they equal? Unequal? What could you have done before even trying to get numerical results to decide whether the two paths would be expected to yield the same answer?

[2.] Find the work done by the force $\vec{F} = x\,\hat{\imath} + y\,\hat{\jmath}$. as one moves along the semicircle of diameter 2 centered at the origin starting from $(1,0)$ and ending at $(-1,0)$. How much work is done in returning to the starting point along the diameter? What does this suggest about the force? Verify your conjecture.

[3.] Show that the area enclosed by a counterclockwise curve $\mathcal{C}$ in the plane is given by

$$\mathcal{A} = \frac{1}{2} \oint_{\mathcal{C}} \left(x\,dy - y\,dx \right)$$

Verify the formula works for the triangle with vertices $(1,0)$, $(0,2)$, $(-1,0)$.

[4.] Compute

$$\int_0^{2\pi} \frac{d\,\theta}{13 + 5\sin\theta}$$

[5.] Compute

$$\int_0^{\infty} \frac{x^2\,dx}{x^4 + 16}$$

This problem closely parallels one of the examples in class. Nevertheless, explain independently the various logical steps you go through, e.g. to get a closed contour to which you can apply the ideas of complex integration. Can you do this problem by a method used in one of your Math 21 classes, such as a trigonometric substitution?

[6.] The "Fermi function" $f(E) = 1/(e^{\beta E} + 1)$ plays a central role in the description of electrons in solids. It gives the number of electrons in a state of energy E if the system has temperature $k_{\rm B}T = 1/\beta$. ($k_{\rm B}$ is Boltzmann's constant.) If E is allowed to be a complex number, locate the poles of $f(E)$.

Extra credit (and very tricky): Evaluate $(1/\beta)\sum_n 1\,/\,(i\omega_n - E)$. Here $\omega_n = \pi(2n+1)/\beta$ are the "Matsubara frequencies" and n are all the integers. This is a very important identity you will learn about in a 2nd year solid state physics grad course. It is very advanced stuff.