PHYSICS 104A, FALL 2016
MATHEMATICAL PHYSICS

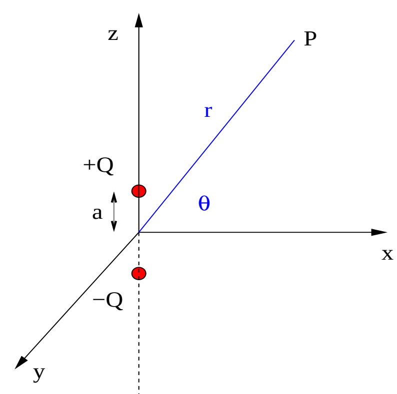
Assignment Three, Due Friday, October 14, 5:00 pm.

[1.] The "Fermi function" $f(E) = 1/(e^{\beta E} + 1)$ plays a central role in the description of electrons in solids. It gives the number of electrons in a state of energy E if the system has temperature $k_B T = 1/\beta$. (k_B is Boltzmann's constant.) If E is allowed to be a complex number, locate the poles of $f(E)$.

Extra credit (and very tricky): Evaluate $(1/\beta) \sum_n 1 / (i\omega_n - E)$. Here $\omega_n = \pi(2n+1)/\beta$ are the "Matsubara frequencies" and n are all the integers. This is a very important identity you will learn about in a 2nd year solid state physics grad course. It is very advanced stuff.

[2.] Compare the relativistic energy $E = mc^2(1 - v^2/c^2)^{-1/2}$ to the classical energy $\frac{1}{2}mv^2$.

[3.] Write down the exact expression for the electric potential $V(r, \theta)$ of a dipole (a pair of charges $+Q$ and $-Q$ located at $(0, 0, +a)$ and $(0, 0, -a)$. See Figure. Expand this expression up to and including order $1/r^4$ for large distances $r >> a$ from the origin.



[4.] When you do "mean field theory" to determine the critical temperature T_c at which a material becomes magnetic, you need to solve the transcendental equation, $m = \tanh(Jm/T)$, Here m is the magnetization and J is called the 'exchange parameter'. Draw pictures of $\tanh(Jm/T)$ for $T = 2.0\,J, 1.3\,J$, and $0.7\,J$. What can you say about these curves intersecting a line of slope one, that is, the nature of solutions to $m = \tanh(Jm/T)$? Use a series expansion of tanh to prove that $m = 0$ is the only solution for $T > J$, but that there are 'nonzero magnetization' solutions with $m \neq 0$ when $T < J$, and that, therefore, the critical temperature $T_c = J$.

[5.] Show that the root mean square distance from the origin after N steps of a random walk in one dimension with probability $p = 1/2$ of going one step to the left and probability $q = 1/2$ of going one step to the right is equal to $\sqrt{N}$. Suppose someone tells you that radiation in the ocean from the Fukushima meltdown is 100 miles from the power plant after 3 months. If the spreading were random (it's not!) how far would you expect the radiation to be after 30 months?