PHYSICS 104A, FALL 2015
MATHEMATICAL PHYSICS

Assignment Three, Due Friday, October 16, 5:00 pm.

[1.] In class we solved the problem of two *equal* masses $m_1 = m_2$ connected by a spring of force constant k. Generalize the solution to $m_1 \neq m_2$. Write equations for $x_1(t)$ and $x_2(t)$ given $x_1(0)$, $x_2(0)$, $v_1(0)$, and $v_2(0)$.

[2.] The vector $\vec{w}$ is the reflection of the vector $\vec{v}$ about the line $y = x/2$. What is the matrix $\mathcal{M}$ which gives $\vec{w} = \mathcal{M}\vec{v}$? Can you generalize the result to any line $y = mx$ through the origin?

[3.] A projection matrix $\mathcal{P}$ is a matrix which satisfies $\mathcal{P}^2 = \mathcal{P}$. What is the matrix $\mathcal{P}$ which projects vectors $\vec{v}$ onto the line $y = x/2$. What are the eigenvalues of your matrix? Can you generalize the result to any line $y = mx$ through the origin?

[4.] Prove that the eigenvalues of *any* projection matrix must be $\lambda = 0, 1$.

[5.] Solve for the eigenfrequencies and eigenvectors (i.e. the normal modes) of four equal masses connected by three springs (i.e. with open boundary conditions).

[6.] Solve for the eigenfrequencies and eigenvectors (i.e. the normal modes) of four equal masses connected by four springs with periodic boundary conditions. (You can do this from scratch, or you can just apply the general formulae for arbitrary N to the special case $N = 4$.)