PHYSICS 104A, FALL 2018
MATHEMATICAL PHYSICS

Assignment Two, Due Friday, October 12, 5:00 pm.

[1.] Compute $\cos(i\pi)$.

[2.] A special case of problem 5 of assignment 1 is that multiplication by i rotates a complex number by $90°$ without changing it's length. Use this idea in the following problem: Let $z = Re^{i\omega t}$ describe the displacement from the origin of a particle at time t. Show that the particle travels in a circle of radius R at velocity $v = R\omega$ and with acceleration of magnitude v^2/R directed towards the center of the circle. Bonus: Find the grammar mistake in this problem.

[3.] For functions of a real variable $df(x)/dx$, for $f(x) = 1/x$, is $-1/x^2$. Compute $df(z)/dz$, for $f(z) = 1/z$. Is it equal to $-1/z^2$? Are the Cauchy-Riemann conditions satisfied?

[4.] For functions of a real variable $df(x)/dx$, for $f(x) = \sqrt{x}$, is $1/(2\sqrt{x})$. Compute $df(z)/dz$, for $f(z) = \sqrt{z}$. Is it equal to $1/(2\sqrt{z})$? Are the Cauchy-Riemann conditions satisfied?

[5.] Is the function $f(z) = z^*$ analytic? (That is, does it satisfy the Cauchy-Riemann conditions?)

[6.] In class we computed the work done by the force $\vec{F} = 4x\,\hat{i} - 2y\,\hat{j}$ in moving a particle from $(0,0)$ to $(1,2)$ along the two paths $y = 2x$ and $y = 2x^2$. We got the same answer. Using the same force, compute the work in going from $(-1,0)$ to $(2,1)$
(a) Along the path $y = (x+1)/3$;
(b) Along the path $y = x^2 - 2x/3 - 5/3$.
First show that both paths contain the desired endpoints!

[7.] You are told the x component of a force $\vec{F}$ is $F_x = 3xy$. You are also told that $\vec{F}$ is conservative, i.e. the work done by $\vec{F}$ in moving between two points in the xy plane does not depend on the path. What can you say about F_y?

[8.] The more complete analysis of forces and their path dependence uses *Green's Theorem*,

$$\oint F_x dx + F_y dy = \iint \big(\partial F_x/\partial y - \partial F_y/\partial x\big) dx dy$$

Here the integral on the left is around a *closed loop* in the xy plane and the integral on the right is over the *area enclosed by the loop*. Why is this equivalent to the statement from class that if $\partial F_x/\partial y = \partial F_y/\partial x$ the integral between two points is path independent?
This equation contains a lot of fancy looking symbols, but the problem you are asked to do is surprisingly simple. Without doing any double integrals, you should be able to say something very quickly about the right hand side if $\partial F_x/\partial y = \partial F_y/\partial x$. Then you need to figure out what an integral around a closed loop has to do with integrals along different paths between two points. Think about what happens to integrals when you change the direction with which you travel from beginning point to ending point.