PHYSICS 104A, FALL 2015
MATHEMATICAL PHYSICS

Assignment Two, Due Friday, October 9, 5:00 pm.

[1.] Compute the potential $V(r, \theta)$ of a uniform disk of total charge Q and radius R which lies in the xy plane with center at the origin.

[2.] List all the possible final locations, and their probabilities, after a random walk in one dimension with $N = 8$ and probability $p = 0.4$ of going one step to the left and probability $q = 0.6$ of going one step to the right. Verify that, on average, the final location is $N(q-p) = 1.6$ steps to the right of the starting point.

[3.] Show that the root mean square distance from the origin after N steps of a random walk in one dimension with probability $p = 1/2$ of going one step to the left and probability $q = 1/2$ of going one step to the right is equal to $\sqrt{N}$. Suppose someone tells you that radiation in the ocean from the Fukushima meltdown is 100 miles from the power plant after 3 months. If the spreading were random (it's not!) how far would you expect the radiation to be after 30 months?

[4.] In understanding the behavior of spin-1/2 particles in quantum mechanics, you will encounter the 2×2 'Pauli matrices'

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In particular, you will need to compute their exponentials

$$A = e^{-it\sigma_x} \qquad B = e^{-it\sigma_y} \qquad C = e^{-it\sigma_z}$$

Using the definition

$$e^M = 1 + M + \frac{1}{2}M^2 + \frac{1}{6}M^3 + \cdots$$

compute A, B and C.

[5.] Compute the inverses of the Pauli matrices.

[6.] The 'commutator' of two matrices A and B is symbolized by $[A, B]$ and is defined by

$$[A, B] = AB - BA$$

Show that the 'spin' matrices

$$S_x = \frac{\hbar}{2}\sigma_x \qquad S_y = \frac{\hbar}{2}\sigma_y \qquad S_z = \frac{\hbar}{2}\sigma_z$$

obey

$$[S_x, S_y] = i\hbar S_z \qquad [S_y, S_z] = i\hbar S_x \qquad [S_z, S_x] = i\hbar S_y$$