PHYSICS 104A, FALL 2018
MATHEMATICAL PHYSICS

Assignment One, Due Friday, October 5, 5:00 pm.

[1.] You can decompose a real number into the sum of two other real numbers in many possible ways: 7=2+5 but also 7=4+3, for example. Prove that the decomposition of a complex number into real and imaginary parts is unique.

[2.] We solved the damped harmonic oscillator in class. Do the same to find the charge $Q(t)$ on the capacitor plate in an LRC circuit. How do you incorporate the initial conditions $Q(t=0)$ and $I(t=0)$ into your general solution?

[3.] Suppose you had an anharmonic spring $F = -kx - ax^3$. What happens if you try a solution $x(t) = e^{i\omega t}$ in the equation of motion? Is there any analytic method which will solve this problem?

[4.] Compute $(0.6 + 0.8i)^{90}$.

[5.] Consider a complex number z as a vector. Describe in words what happens to z if you multiply it by $\rho\, e^{i\phi}$. (Here ρ and ϕ are real numbers.) How does the length of $\rho\, e^{i\phi}\, z$ compare to that of z? What about its direction?

[6.] The ground state energy of a one dimensional metallic chain with N atoms is given by

$$E_0 = \sum_{j=-N/4}^{N/4} -2t\cos\frac{2\pi j}{N}$$

Here the parameter t is the 'hopping integral' and is determined by the overlap of wavefunctions on adjacent atoms. Depending on the material, t might have a value of, for example, $t = 2.5$ eV for polyacetylene [1]. Find a closed-form expression for this sum. Can you discover what it converges to in the "thermodynamic limit" $N \to \infty$?

[7.] In class we discussed the solutions of $z^4 = 1$ being $z = 1, i, -1, i$. Find (a) the solutions to $z^4 = 1 + i$; and (b) the solutions to $z^4 - (3+i)z^2 + 2 + i = 0$.

[8.] Compute $\ln(3 + 4i)$.

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[1] The classic paper on polyacetylene, pointing out the existence of solitons (!), is here: http://journals.aps.org/prl/pdf/10.1103/PhysRevLett.42.1698. We will derive the key results of this paper when we go through our discussion of matrices and eigenvalues.