# PHYSICS 104A, FALL 2015
# MATHEMATICAL PHYSICS

**Assignment One, Due Friday, October 2, 5:00 pm.**

[1.] We solved the damped harmonic oscillator in class. Do the same to find the charge $Q(t)$ on the capacitor plate in an LRC circuit. How do you incorporate the initial conditions $Q(t=0)$ and $I(t=0)$ into your general solution?

[2.] Suppose you had an anharmonic spring $F = -kx - ax^3$. What happens if you try a solution $x(t) = e^{i\omega t}$ in the equation of motion? Is there any analytic method which will solve this problem? (We will see how to deal with anharmonicities easily in Physics 102!)

[3.] Compute $(0.6 + 0.8i)^{90}$.

[4.] The ground state energy of a one dimensional metallic chain with $N$ atoms is given by

$$E_0 = \sum_{j=-N/4}^{N/4} -2t \cos \frac{2\pi j}{N}$$

Here the parameter $t$ is the 'hopping integral' and is determined by the overlap of wavefunctions on adjacent atoms. Depending on the material, $t$ might have a value of, for example, $t = 2.5$ eV for polyacetylene [1]. Find a closed-form expression for this sum. Can you discover what it converges to in the "thermodynamic limit" $N \to \infty$?

These next three problems illustrate how power series expansions arise in some important situations. You have probably done [5.] before in Physics 9D. Meanwhile, [6.] and [7] are key steps in the analysis of magnetic ordering in solids.

[5.] Compare the relativistic energy $E = mc^2(1 - v^2/c^2)^{-1/2}$ to the classical energy $\frac{1}{2}mv^2$.

[6.] When you do "mean field theory" to determine the critical temperature $T_c$ at which a material becomes magnetic, you need to solve the transcendental equation, $m = \tanh(Jm/T)$, Here $m$ is the magnetization and $J$ is called the 'exchange parameter'. Draw pictures of $\tanh(Jm/T)$ for $T = 2.0\,J, 1.3\,J$, and $0.7\,J$. What can you say about these curves intersecting a line of slope one, that is, the nature of solutions to $m = \tanh(Jm/T)$? Use a series expansion of tanh to prove that $m = 0$ is the only solution for $T > J$, but that there are 'nonzero magnetization' solutions with $m \neq 0$ when $T < J$, and that, therefore, the critical temperature $T_c = J$.

[7.] (Extra credit) Show that $m(T) \sim (T - T_c)^{\frac{1}{2}}$ for $T$ just a bit below $T_c$. The power 1/2 is the "mean field critical exponent" of the magnetic phase transition.

---

[1] The classic paper on polyacetylene, pointing out the existence of solitons (!), is here: http://journals.aps.org/prl/pdf/10.1103/PhysRevLett.42.1698 We will derive the key results of this paper when we go through our discussion of matrices and eigenvalues.