We are going now to compute this sum:

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{x^N}{N!}$$

It looks like the exponential that we just did, except with only the odd powers and also with some minus signs sprinkled in.

```
/* this is a program to do a second weird sum*/
#include <stdio.h>
#include <math.h>
int main(void)
{
double prod=1, sum=0, x;
int j, N;
printf("Enter N and x");
printf("\n");
scanf("%i %lf",&N,&x);
for (j=1; j<N; j=j+2)
{
sum=sum+pow(-1,j/2)*pow(x,j)/prod;
printf("\n   %i",j);
printf("  ");
printf("%20.10lf",sum);
prod=prod*(j+1);
prod=prod*(j+2);
}
printf("\n");
return 0;
}
```

[1] It is important to understand integer arithmetic in looking at this program. When you do arithmetic with integers, C throws away whatever would occur after the decimal point. Thus $1/2 = 0$, 3/2=1, 4/3=1, 9/4=2, etc. Why is this important to the operation of the program?

[2] This code is sort of tricky in other ways too! Step through the loop "manually" to make sure it is doing the right thing. This is a very useful way to write and verify codes!

[3] Run your program with $N = 10$ and $x = 1$. Do you recognize the number you are getting? Probably not! Use a calculator to compute sin(1 radian). What do you notice? Run your program with $N = 10$ and $x = 0.5$ and compare with sin(0.5 radian). Run your program with $N = 10$ and $x = 0.8$ and compare with sin(0.8 radian).

[4] So here's the amazing fact: If N is large enough,

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Just as with the exponential we see that a trigonometric function can be written as a polynomial of very high (infinite) order.

[5] Write a program to do this sum:

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots$$

and verify you get cos(x).

[6] If you are really into complex numbers, use these equations you learned for e^x, sin(x), cos(x) to prove de Moivre's theorem:

$$e^{ix} = \cos(x) + i\sin(x)$$

This equation tells you that $e^{i\pi} = -1$! Who knew that e and π had anything to do with each other!