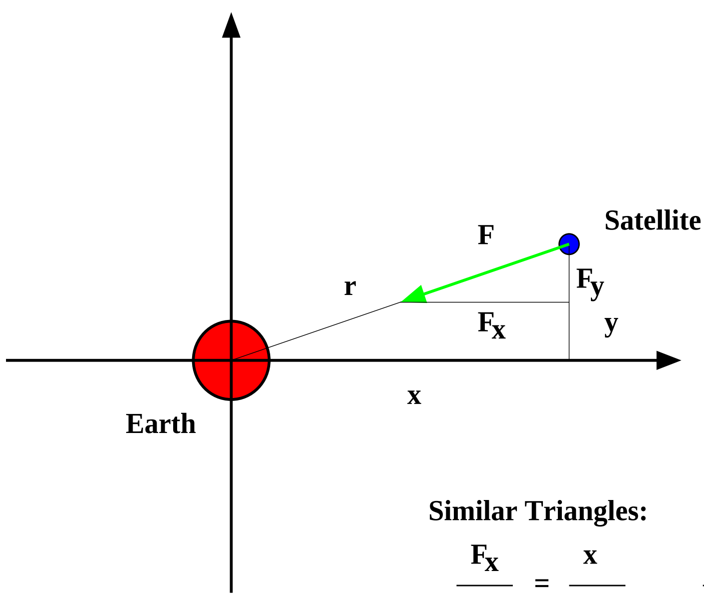
The Molecular Dynamics Method II: Satellite Motion

In MD-1 you learned the basics of Molecular Dynamics (MD). Here we will do a problem in two dimensions using the gravitational force law (instead of a spring force). To allow motion in 2D we need to keep track of two coordinates x, y, as well as the velocities in the x and y directions: v_x and v_y

Newton's law of gravity says that the force between any two masses M and m has size $F = GMm/r^2$ were r is the distance between the masses and G is a constant whose value is $G = 6.67 * 10^{-11}$ (if masses are measured in kilograms and r in meters). The direction of the force is along the line connecting the masses and is attractive.

We will consider the motion of a satellite (mass m) around the earth (mass M). Since the earth is so big, we will assume it is stationary at the origin. A little geometry then allows us to get the pieces (components) of the force in the x and y directions:



Similar Triangles:

$$\frac{F_x}{F} = \frac{x}{r} \qquad \frac{F_y}{F} = \frac{y}{r}$$

Actually need to add minus sign to F_x and F_y to account for direction (attractive)

So now the heart of an MD code for a satellite going around the earth is this:

```
x=x+vx*dt
y=y+vy*dt
r=sqrt(x*x+y*y)
vx=vx-(G*M*x/r^3)*dt
vy=vy-(G*M*y/r^3)*dt
t=t+dt
```

Why did the $1/r^2$ in our formula for F become x/r^3 ? (Hint: See the Figure and the comment on similar triangles.) What happened to the mass of the satellite m?

Your challenge is write a complete MD code for this problem. One thing you will need to do is look up the mass of the earth. You will also need to figure out some reasonable initial conditions which will keep the satellite from escaping the earth completely or crashing to the surface of the earth. A fact that might help you is that a circular orbit of radius r around a mass M is achieved with a velocity $v = \sqrt{GM/r}$.

The time it will take a satellite to do one circular orbit of radius r is the distance travelled, $d = 2\pi r$, divided by the velocity $v = \sqrt{GM/r}$. Use this fact to compute the radius of a *geosynchronous orbit* (one whose period T is equal to one day). Check your answer against the value given by wikipedia.

A tricky point concerns how big dt can be. In MD-1 we emphasized that dt had to be "small". This was actually an imprecise statement, because we really need to specify "small compared to what"? The answer is "small compared to the period of the motion": in this case, the time it takes the satellite to circle the earth. So if you do a geosynchronous orbit, dt should be small compared to one day ($T = 86,400$ seconds). Thus $dt = 10$ or even $dt = 100$ is "small" compared to 86,400.

Challenge! The astronomer Kepler, in accumulating data for the distances of planets from the sun and their periods, noticed an amazing pattern: when he computed r^3/T^2 he got nearly the same number for all the planets, even though their radii r and periods T were wildly different. First, verify this statement is true by completing the following table:

Planet	Distance from Sun R (miles)	Length of Year T (seconds)	$\Rightarrow R^3/T^2$
Mercury	36,000,000	8,000,000	808,000,000
Venus	67,000,000	19,400,000	800,000,000
Earth	93,000,000	31,500,000	811,000,000
Mars	140,000,000	59,000,000	
Jupiter	480,000,000	370,000,000	
Saturn	890,000,000	930,000,000	
Uranus	1,800,000,000	2,700,000,000	
Neptune	2,800,000,000	5,200,000,000	
Pluto	3,650,000,000	7,800,000,000	

I have used the units of miles and seconds in the table above. Get the constant value instead if you use meters and seconds. Then **prove** Kepler's rule from the discussion relating the radius, velocity, and period of circular orbits and get a formula for the constant. Does it agree with observations of the solar system? (Remember to use the mass of the sun in the formula you derive, not the mass of the earth!)

It is important to close this discussion by emphasizing that Kepler's observations were what led Newton to his formula $F = GMm/r^2$ for the force of gravity. Newton realized that a $1/r^2$ formula (and no other!) would make r^3/T^2 constant and allow him to explain Kepler's data. This is an interesting illustration of how science works.