

C PROGRAMMING: SUMMING GEOMETRIC SERIES

The Greek mathematician Zeno formulated the following paradox: Your Cosmos R.A. is running a race. In order to finish, she first has to run $1/2$ the distance. Then she must run $1/2$ the remaining distance, then $1/2$ of what is left again. This process goes on indefinitely: You can keep dividing the remaining distance by two an infinite number of times. Hence the R.A. can never finish the race.

Does anyone know what is wrong with Zeno's argument? We can understand how to answer Zeno a bit better by learning about **geometric series**.

A **geometric series** is a collection of numbers which begins with a real number A and then is multiplied repeatedly by a second real number r . *ie*

$$A, Ar, Ar^2, Ar^3, \dots$$

Just as when we considered arithmetic series, we will learn to sum up the elements of a geometric series, *ie* compute:

$$\mathcal{S} = A + Ar + Ar^2 + Ar^3 + Ar^4 + \dots + Ar^N$$

We will figure out how to do it with pencil and paper first. The trick is to take the original \mathcal{S} and multiply by r :

$$\begin{aligned} \mathcal{S} &= A + Ar + Ar^2 + Ar^3 + Ar^4 + \dots + Ar^N \\ r\mathcal{S} &= Ar + Ar^2 + Ar^3 + Ar^4 + \dots + Ar^{N+1} \end{aligned}$$

If we subtract all sorts of things cancel!

$$(1 - r)\mathcal{S} = A - Ar^{N+1}$$

Therefore,

$$\mathcal{S} = \frac{A - Ar^{N+1}}{1 - r}$$

Notice that if $r < 1$ and we make $N = \infty$ we get $\mathcal{S} = A/(1 - r)$ because any number less than one raised to a very high power is zero.

We can now answer Zeno: Even if we allow the sum $\mathcal{S} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ to go on forever, the result is not infinite. In fact, according to our formula, $\mathcal{S} = 1$. Your cosmos TA will finish her race.

Let's verify this formula by writing a program to sum a geometric series.

```

/* this is a program sums a geometric series*/
#include <stdio.h>
#include <math.h>
int main(void)
{
int j,N;
double sum=0,A,r;
printf("Enter A");
printf("\n");
scanf("%lf",&A);
printf("Enter r");
printf("\n");
scanf("%lf",&r);
printf("Enter N");
printf("\n");
scanf("%i",&N);
for (j=0; j<N; j=j+1)
{
sum=sum+A*pow(r,j);
}
printf("%12.6lf \n",sum);
return 0;
}

```

Comments:

[1] Not much to say here really about the code! How about running the code for $A = 0.5, r = 0.5, N = 20$ to check out our answer to Zeno, namely that,

$$\mathcal{S} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 1$$

[2] Try also some other choice like $A = 1.7, r = 0.3, N = 20$ and verify your code agrees with the pencil and paper formula.