

C PROGRAMMING: COMPUTING FACTORIALS

We wrote a program to compute the sum of the first N integers. How about this for the product of the first N integers:

```

/* this is a program to multiply the first N integers*/
#include <stdio.h>
#include <math.h>
int main(void)
{
int prod=1;
int j, N;
printf("Enter N");
printf("\n");
scanf("%i",&N);
printf("  j                j!  ");
for (j=1; j<N; j=j+1)
{
prod=prod*j;
printf("\n   %i",j);
printf("  ");
printf("%20i",prod);
}
printf("\n");
return 0;
}

```

Comments:

[1] What is the product of the first N integers called? Answer: $N!$, read ‘ N factorial’.

[2] In what context(s) have you encountered factorials? Possible answer: binomial coefficients. If we define

$$\binom{N}{k} = \frac{N!}{(N-k)! k!}$$

then

$$(a+b)^N = \sum_{k=0}^N \binom{N}{k} a^{N-k} b^k$$

Note that $0!$ is defined to be one. We will use factorials quite a bit in the coming weeks.

[3] **In-class Exercise One:** Evaluate $\binom{3}{2}$, $\binom{7}{4}$, and $\binom{5}{0}$.

For the last one you will need to use $0! = 1$.

[4] **In-class Exercise Two:** Compute

$$\binom{2}{0} + \binom{2}{1} + \binom{2}{2}.$$

Compute

$$\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3}.$$

Compute

$$\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}.$$

What pattern do you notice? Can you prove it?!