CHAOS AND THE LOGISTIC MAP

This is an assignment you can do if you find yourself getting ahead in class.

The word 'chaos' has a colloquial meaning of 'very disorganized' and 'messy'. It also is used to describe a specific set of properties of certain equations: Instead of smoothly evolving to some steady final value, the solution instead jumps around a lot and never settles down. Furthermore, the behavior depends on how you start the system out. Even a small change in the initial conditions can complelely alter the outcome. A very simple equation which exhibits these phenomena is the "logistic map."

In the logistic map you start with an initial value x_0 and get x_1 by:

$$x_1 = 1 - A\,x_0^2 \;.$$

The number A is a constant (more about it below). Then you get x_2:

$$x_2 = 1 - A\,x_1^2 \;.$$

More generally, if you know x_n you get the next value x_{j+1} via:

$$x_{j+1} = 1 - \; A x_j^2 \;.$$

[1.] Write a code to compute the first N values of x_j.

[2.] Run your code for $A = 0.5$ and the three initial values $x_0 = 0.25, 0.50, 0.75$. What happens? Make a plot of x_j versus j for these three different starting points.

[3.] Do the same things for $A = 0.2$ and $A = 0.6$ and $A = 0.7$. Is there any difference in what happens?

[4.] Now run your code for $A = 1.1$ and $A = 1.3$ Is the behavior changed? Make a plot of x_j versus j for these two cases.

[5.] Finally, run your code for $A = 1.5$ and the initial value $x_0 = 0.25$. What happens? Make a plot of x_j versus j.

What you should find is that for the smallest values of A, the stream of x_j approaches a constant at large j, and the constant doesn't depend on the starting point x_0. When A gets larger, though, you start oscillating between two values. Then when A is even larger, you bounce around between four values. Ultimately, for A big enough, you do not have any pattern for the x_j: they just skip around "chaotically".

The message is that there are simple equations the basic nature of the solutions of which change completely as one tunes a parameter (in this case the number "A"). The same sort of thing happens in much more complicated "real life" situations: the equations which govern the flow of air around a car or airplane have solutions which are nice and smooth at low velocities, but become chaotic and turbulent when the velocity is higher.